Lecture notes

Materials Fabrication (McMaster University)

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Summary of Eng 4J03

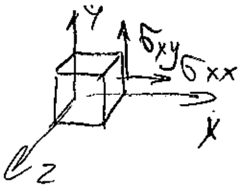
1. Concept of the stress tensor of II-nd rank

2. Properties of σ_{ij}

a) physical meaning of components

$$\sigma_{xx} \cdot dz \cdot dy = f_{xx} \quad \text{or} \quad \sigma_{xy} \cdot dz \cdot dy = f_{xy}$$

b) σ_{ij}



σ_{ij} - is associated with two axes of reference system

c). equilibrium conditions:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0$$

d). equilibrium of momentum: $\sigma_{ij} = \sigma_{ji}$

e). transformation law: $\sigma'_{ij} = a_{ik} a_{jL} \sigma_{KL}$

f). Relation between stress vector and stress tensor:

$$\overline{\sigma_i} = \sigma_{ij} \cdot \overline{n_j}$$

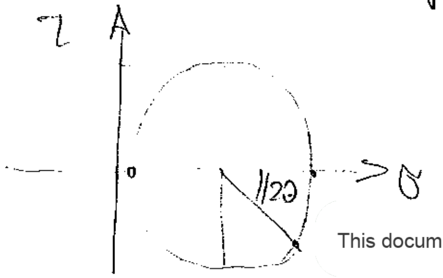
3. Transformation of the II-nd rank tensor.

a) transformation matrix

		OLD	
		X	Y
NEW	X'	cos X,X'	cos∡(X',Y)
	Y'		

b) Decomposition of stress tensor into deviatoric part and axials

4. Mohr circle for stresses and how to use it



$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta = - \frac{(\sigma_x - \sigma_y)}{}$$

5. Evaluation of principal axis system

a). finding principal stresses by matrix diagonalization method i.e

$|\sigma_{xy} - \lambda I| = 0$ $\lambda_1, \lambda_2, \lambda_3$ - are principal stresses

$$\det \begin{vmatrix} \sigma_{xx}-\lambda & \sigma_{xy} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy}-\lambda & \sigma_{zx} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz}-\lambda \end{vmatrix} = 0 \qquad \underline{\lambda^3 - I_1\lambda^2 - I_2\lambda - I_3 = 0}$$

I_1, I_2, I_3 are invariants.

$$I_1 = \sigma_x + \sigma_y + \sigma_z = \text{trace}(\sigma_{ij})$$

$$I_2 = -\sigma_{xx}\sigma_{yy} - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} + \sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2$$

b). find orientation of the principal axis system

$\sigma_{ij} \cdot \bar{X} = \bar{X} \cdot \lambda$ $\bar{X} = [l, m, n]$ λ_i - principal stresses

l_i, m_i, n_i - direction cosine of the i-th principal axis

$$(\sigma_{xx} - \lambda_i)\, l_i + \tau_{yx}\, m_i + \tau_{zx} \cdot n_i = 0$$

$$\tau_{xy} \cdot l_i + (\sigma_{yy} - \lambda_i)\, m_i + \tau_{zy} \cdot n_i = 0$$

$$\tau_{xz} \cdot l_i + \tau_{yz}\, m_i + (\sigma_{zz} - \lambda_i)\, n_i = 0$$

$$\boxed{l_i^2 + m_i^2 + n_i^2 = 1}$$

6. The strain tensor.

a). physsical interpretation of strain tensor components i.e. e_{xx}, e_{xy} etc

$$e'_{xx} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta .$$

b). properties of the strain tensor

- meaning of the components
- decomposition of the strain tensor into symmetrical and antisymmetrical part

$$e_{xy} = \underbrace{\frac{1}{2}\left(e_{xy}+e_{yx}\right)}_{\varepsilon_{xy}} + \underbrace{\frac{1}{2}\left(e_{xy}-e_{yx}\right)}_{\omega_{xy}}$$

- transformation law for tensors of II-nd rank:

$$\boxed{e'_{ij} = a_{ik}\,a_{jl}\,e_{kl}}$$

7. Strain gauge theory

a). calculation of principal axes system from straing gauge arrangement for strains and stresses

$$\varepsilon'_{xx} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\sigma_{max} = \frac{E}{1-\nu^2}\left(\varepsilon_{max} - \nu\,\varepsilon_{min}\right) \qquad \tau_{max} = \gamma_{max}\,\frac{E}{2(1+\nu)}$$

$$\sigma_{min} = \frac{E}{1-\nu^2}\left(\varepsilon_{min} + \nu\,\varepsilon_{max}\right)$$

8. Stress - strain relations for elastic deformat of Isotropic materials

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right) \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz} = 2\varepsilon_{yz}$$

$$\varepsilon_y = 1/E\left(\sigma_y - \nu(\sigma_x + \sigma_z)\right) \qquad \gamma_{xz} = \frac{1}{G}\tau_{xz} = 2\varepsilon_{xz}$$

$$\varepsilon_z = 1/E\left(\sigma_z - \nu(\sigma_x + \sigma_y)\right) \qquad \gamma_{xy} = 1/G\,\tau_{xy} = 2\varepsilon_{xy}$$

$$E = \frac{1}{S_{11}} = C_{11} \qquad G = \frac{1}{S_{44}} = C_{44}$$

a) Hooks Laws for isotropic materials

$$\sigma = E\varepsilon \qquad \tau = G\gamma \qquad p = B \cdot \Delta$$

9. Yield Criteria

a) Tresca : $\sigma_{max} - \sigma_{min} \geq Y$ $\quad \tau_{max} \geq Y/2 \geq k$

b) von Mises $\frac{1}{\sqrt{2}}\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2} \geq Y$

c) geometrical interpretation

10. Plastic stress - strain relations i.e. flow rules

$$E_{PL} = \frac{\bar{\sigma}}{d\bar{\varepsilon}}$$

a) $d\varepsilon_1 = \frac{1}{E_{PL}} \cdot \left(\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)\right]$

$d\varepsilon_2 = \frac{1}{E_{PL}} \cdot \left[\sigma_2 - \frac{1}{2}(\sigma_1 + \sigma_3)\right]$

$d\varepsilon_3 = \frac{1}{E_{PL}} \cdot \left(\sigma_3 - \frac{1}{2}(\sigma_1 + \sigma_2)\right]$

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$$

$$d\varepsilon_1 + d\varepsilon_2 + d\varepsilon_3 = 0$$

10. Flow rules - continuation

b) $\overline{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)+(\sigma_2-\sigma_3)^2}$

$\overline{\varepsilon} = \sqrt{\frac{2}{3}(\varepsilon_1^2+\varepsilon_2^2+\varepsilon_3^2)}$

c).

$$d\varepsilon_1 : d\varepsilon_2 : d\varepsilon_3 = \left[\sigma_1 - \frac{1}{2}(\sigma_2+\sigma_3)\right] : \left[\sigma_2 - \frac{1}{2}(\sigma_1+\sigma_3)\right] : \left[\sigma_3 - \frac{1}{2}(\sigma_1+\sigma_2)\right]$$

ability to calculate stress and strain ratios during deformation

11 Instability under :

a). uniaxial tension : $\frac{d\overline{\sigma}}{d\overline{\varepsilon}} = \sigma$ $\underline{n = \varepsilon}$

b). balanced biaxial tension :

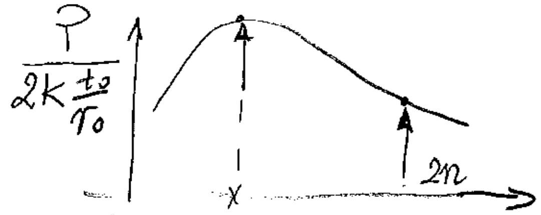
$\frac{d\overline{\sigma}}{d\overline{\varepsilon}} = \frac{\overline{\sigma}}{2}$ $\overline{\varepsilon} = 2n$

c). instability of the pressure ~~during~~ in the spherical pressure vessel

$\frac{d\overline{\sigma}}{d\overline{\varepsilon}} = \frac{3}{2}\overline{\sigma}$ $\overline{\varepsilon} = \frac{2}{3}n$

$\varepsilon_1 = \varepsilon_2$

$\varepsilon_3 = -2\varepsilon_1 = -2\varepsilon_2$

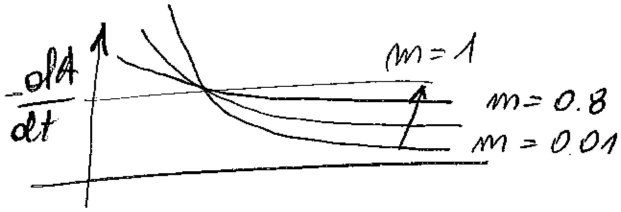


12. Strain Rate and Temperature effects

a). $\sigma = K\varepsilon^{n} \cdot \dot{\varepsilon}^{m}$

b). analysis of superplastic material

$$-\frac{dA}{dt} = \frac{1}{A^{\frac{(1-m)}{m}}}$$



c). analysis of inhomogeneous bar

$$\varepsilon_b = -m \ln\left(1 - f^{1/m}\right)$$

d) temperature dependence of the flow stress:

$$\dot{\varepsilon} = C \cdot \exp\left(-\frac{Q}{RT}\right) \qquad \sigma = \dot{\varepsilon} \cdot \exp\left(\frac{Q}{RT}\right) \equiv Z$$

$$\dot{\varepsilon} = A \cdot \left[\sinh(\alpha\sigma)\right]^{\frac{1}{m}} \cdot \exp\left(-\frac{Q}{RT}\right)$$

- consequences
- calculations of flow stress, m, activation energy etc

e). calculation of temperature rise during deformation

$$w = \int_0^{\bar{\varepsilon}} \bar{\sigma}\, d\bar{\varepsilon} \qquad \Delta T = \frac{\alpha \int \bar{\sigma}\, d\bar{\varepsilon}}{\rho c} = \frac{\alpha (\bar{\sigma})\, \bar{\varepsilon}}{\rho c}$$

ρ - density c - mass heat capacity $\alpha \sim 0.98$

13. Ideal work of uniform energy

$$W_i = \int_0^{\bar\varepsilon} \bar\sigma \, d\bar\varepsilon \qquad W_i = \frac{k\bar\varepsilon^{n+1}}{n+1} \text{ - ideal work}$$

- extrussion pressure:

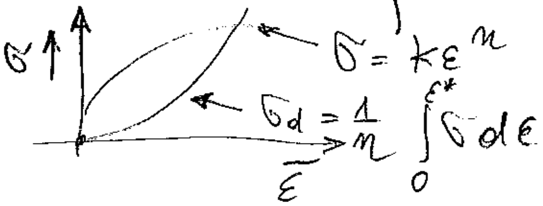
$$P_e \geqslant \int \bar\sigma \, d\bar\varepsilon \qquad W_e = W_i + W_r + W_f$$

- steady state drawing

$$\sigma_d = \frac{1}{n} \int_0^{\varepsilon^*} \bar\sigma \, d\bar\varepsilon \, . \qquad \sigma_d = \frac{1}{n}\left(\frac{k\,\varepsilon^{n+1}}{n+1}\right)$$

$$\varepsilon^* = n(n+1) \text{ - maximum drawing reduction}$$

$$\varepsilon^* = \ln\left(\frac{1}{1-r^*}\right)$$



- optimum die angle during extrussion.

$$\alpha^* \sim \sqrt{\frac{\mu r}{1-r}}$$

α^* - optimum angle
r - reduction
μ - coefficient of friction

14. Slab analysis

a) drawing process (sliding friction) $\mu P < k$

$$\frac{\sigma_d}{\underset{\sim}{2k}} \underset{\bar\sigma}{=} \frac{1+B}{B}\left[1 - \exp(-B\varepsilon_h)\right] \qquad \varepsilon_h = \ln\left(\frac{t_0}{t}\right)$$

$$\varepsilon_h = \ln\left(\frac{1}{1-r}\right)$$

$$B = \mu \cot\alpha \qquad \underline{2k = \text{const}}$$

b) wire drawing - sticking friction $\mu P = k$

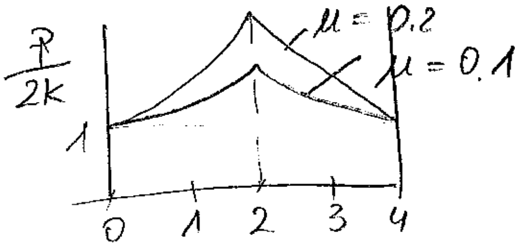
$$\sigma_d = \left(1+\frac{1}{\sin 2\alpha}\right)\cdot 2k\cdot \varepsilon_h$$

a). general idea of equilibrium of forces - how we can derive drawing stress etc.

15. Open Die Forging

a). Pressure and pressure distribution: (sliding friction)

$$\frac{P}{2k} = \exp\frac{2\mu x}{h}$$



$$P_{max} = 2k \exp\frac{\mu b}{h}$$

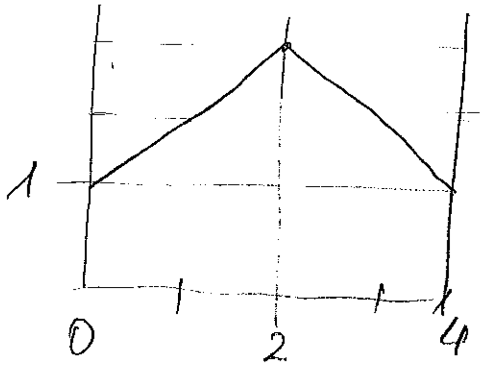
$$P_{av} = \frac{2kh}{\mu b}\left[\exp\left(\frac{\mu b}{h}\right)-1\right] \text{ or } P_{av} = 2k\left[1+\frac{1}{2}\frac{\mu b}{h}\right]$$

b). Sticking friction

$$P = 2k\left[1+\frac{x}{h}\right]$$

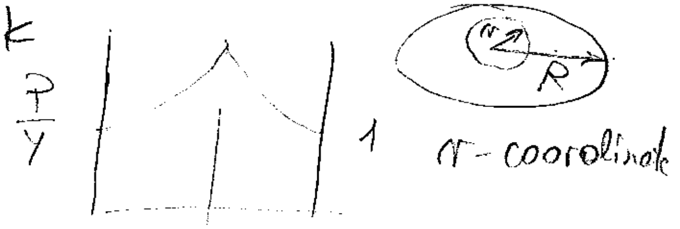
$$P_{max} = 2k\left[1+\frac{b}{2h}\right]$$

$$P_{av} = 2k\left[1+\frac{b}{4h}\right]$$



15. Axisymmetric Compression

a). sliding friction : $\mu P < k$



$$P = Y \exp\left(\frac{2\mu}{h}(R-r)\right)$$

$$P_{max} = Y \exp \frac{2\mu R}{h}$$

$$P_{av} \cong Y\left(1 + \frac{a}{3} + \frac{a^2}{12}\right)$$

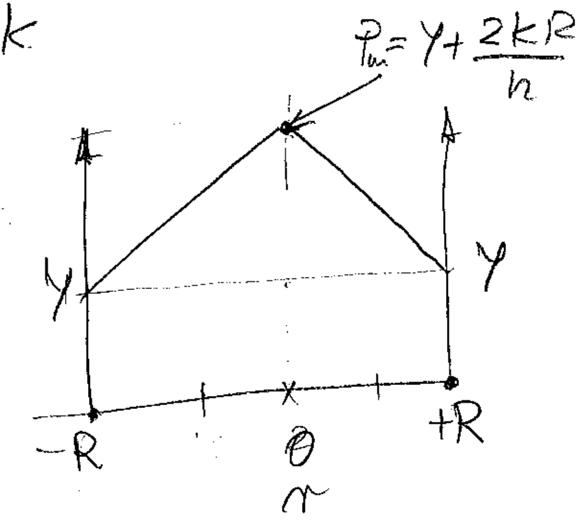
$$a \simeq \frac{2\mu R}{h}$$

b) Sticking friction : $\mu P = k$

$$P = Y + \frac{2k}{h}(R-r)$$

$$P_{max} = Y + \frac{2kR}{h}$$

$$P_{av} = Y + \frac{2kR}{3h}$$

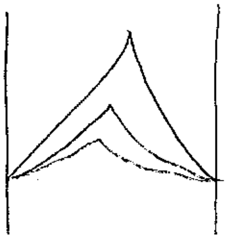


16. Rolling

a). Pressure distribution in entry and exit zone.

$$P_E = Y\left(\frac{h}{h_0}\right) \exp\left[\mu(H_0 - H)\right]$$

$$P_x = Y\left(\frac{h}{h_f}\right) \exp(\mu H)$$



$$L = \sqrt{R\Delta h}$$

$$r = \frac{h_0 - h_f}{h_0} = \frac{\Delta h}{h_0}$$

b). determination of the neutral point

$P_E = P_x$ => from date R, α, h_f

$$H_0 = 2\sqrt{\frac{R}{h_f}}\,\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\,\alpha\right)$$

$$H_n = \frac{1}{2}\left(H_0 - \frac{1}{\mu}\ln\frac{h_0}{h_f}\right)$$

$$\varphi_n = \sqrt{\frac{h_f}{R}}\,\tan\left(\sqrt{\frac{h_f}{R}}\cdot\frac{H_n}{2}\right)$$

c). effect of Front and Bock tension:

$$P_E = \left(Y - \sigma_{bt}\right)\cdot\frac{h}{h_0}\exp\left(\mu\left(H_0 - H\right)\right)$$

$$P_x = \left(Y - \sigma_{ft}\right)\frac{h}{h_f}\exp\left(\mu H\right)$$

d). Rolling Process Parameters :

a). Arerage Pressure :

$$P_{av} = \frac{h}{\mu L}\left(\exp\frac{\mu L}{h} - 1\right)\left(\sigma_0 - \sigma_t\right)$$

b). Roll force $F = L\cdot W\cdot P_{av}$ $L\cdot W$ - contact aree

c). Torque per Roll $T = \frac{FL}{2}$ $L = \sqrt{R\cdot\Delta h}$

d). Rolling Process Parameters (Cont).

d). Power $= T\omega = \left(\frac{FL}{2}\right)\cdot(2\pi N)$

- SI units Power $= \frac{\pi F L N}{60\,000}$ (kW)

F(kN) L(m) N = rpm

- Imperial
Power $= \frac{\pi F L N}{33000}$ (hp)

F(kips) L(ft) N = rpm

e). constrection of Sendzimir roller

17. Bending

a). stress field and strain field in bending

b). calculation of bending moment

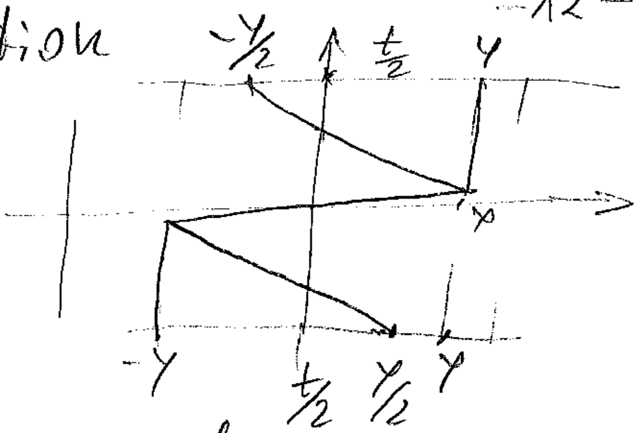
- $M = 2\int_0^{t/2} w\,\sigma_x\, z\,dz$

+ a) elastic
b) plastic $M = 2wY\int_0^{t/2} z\,dz = \frac{wYt^2}{4}$

c). spring back effect

- calculation of the tool and after
spring [illegible] $\frac{3Y}{tE'}$]

d) residual stress distribution



$$\sigma_x' = \sigma_Y - \Delta\sigma_x$$

$$\sigma_x' = Y\left(1 - \frac{3z}{t}\right)$$

e). bending with superimposed tension

- effect of superimposed tension on the stress, strain distribution position of the neutral point.

f). sheet bendability $\frac{R}{t}$

$$\boxed{\frac{R}{t} = \frac{1}{2A_r} - 1}$$

18. Plastic Anisotropy.

a). evaluation of effective stress and strain for hill's normal anisotropy of sheet

$\sigma_3 = 0$

$$\bar{\sigma}^2 = \left(\sigma_1^2 + \sigma_2^2 - \frac{2R}{1+R}\sigma_1\sigma_2\right)$$

$$d\bar{\varepsilon}^2 = \frac{(1+R)^2}{1+2R}\cdot\left(d\varepsilon_1^2 + d\varepsilon_2^2 + \frac{2R}{1+R}d\varepsilon_1\, d\varepsilon_2\right)$$

18. Plastic anisotropy (cont.)

$$d\varepsilon_1 = \frac{d\bar{\varepsilon}}{\bar{\sigma}}\left(\sigma_2 - \frac{R}{1+R}\sigma_1\right)$$

$$d\varepsilon_2 = \frac{d\bar{\varepsilon}}{\bar{\sigma}}\left(\sigma_2 - \frac{R}{1+R}\sigma_1\right)$$

$$d\varepsilon_3 = \frac{d\bar{\varepsilon}}{\bar{\sigma}}\left(-\frac{\sigma_1}{2} - \frac{\sigma_2}{2}\right)$$

b) effect of anisotropy on yield surface

19. Drawing process:

a) stress state during different parts of drawn cup.

NO

b) drawing Force:

$$F_d = 2\pi r_1 t \sigma_f \ln\left(\frac{r}{r_1}\right)$$

$$F_{d,max} = 2\pi r_1 t \sigma_f \ln\left(\frac{r_0}{r_1}\right) = 2\pi r_1 t \sigma_f \ln\left(\frac{d_0}{d_1}\right)$$

c) stress in the wall

$$\sigma_w = \sigma_f \ln\left(\frac{d_0}{d_1}\right)$$

$$\Downarrow$$

d) $LDR = \frac{d_0}{d_1}$

$$\ln(LDR) = \frac{\sigma_w}{\sigma_f}$$

$R \approx 2.1-2.2$ practical

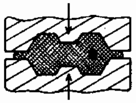
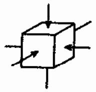
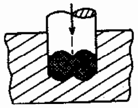
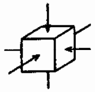
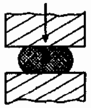
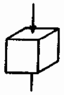
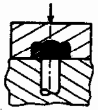
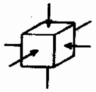
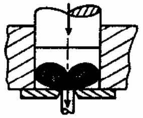
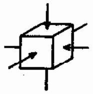
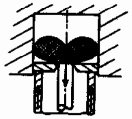
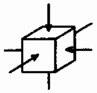
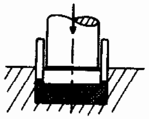
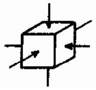
e) ~~η – deformation efficiency~~

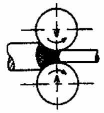
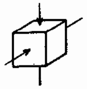
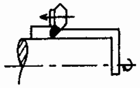
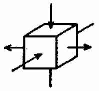
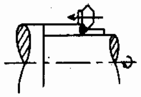
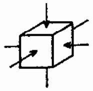
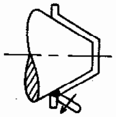
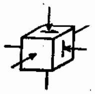
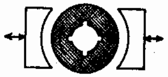
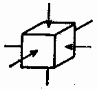
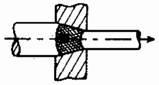
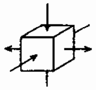
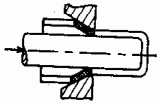
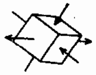
$$\beta = \frac{\sigma_w}{\sigma_f} \qquad \ln(LDR) \simeq \beta$$

$$\ln(LDR) = \eta \beta$$

$$\beta \simeq \sqrt{\frac{R+1}{2}} \qquad \ln(LDR) = \eta \sqrt{\frac{R+1}{2}}$$

$$R = \frac{R_0 + 2R_{45} + R_{90}}{4}$$

Basic Deformation Mechanism	No.	Forming Process		Approximate State of Stress at Square	Instantaneous Zone of Deformation (Cross-Hatched)	State of Deformation
		Name	Diagram			
Squeezing (chipless forming)	1	Closed die forging (drop or press)			Whole part	Nonsteady
	2	Coining			Whole part	Nonsteady
	3	Upsetting (open dies)			Whole part	Nonsteady
		Upsetting (closed dies)			Whole part	Nonsteady
	4	Extruding flats and rounds (direct or forward)			Zone near orifice	Steady
		Extruding flats and rounds (indirect or backward)			Zone near orifice	Steady
	5	Extruding hollow shapes (reverse canning)			Zone under punch	Nonsteady

Basic Deformation Mechanism	*No.*	*Forming Process*		*Approximate State of Stress at Square*	*Instantaneous Zone of Deformation (Cross-Hatched)*	*State of Deformation*
		Name	*Diagram*			
Squeezing, (chipless forming) —*contd.*	6	Rolling flats, rounds and other shapes (without front and back pull)			Zone under rolls	Steady
	7	Spin forging of tubes (drawing type)			Zone under roller	Steady
		Spin forging of tubes (extruding type)			Zone under roller	Steady
	8	Spin forging of cones			Zone under roller	Steady
	9	Swaging and kneeding			Zone under tools	Nonsteady
Drawing (chipless forming)	10	Wire and bar drawing			Zone in die	Steady
	11	Tube drawing			Zone in die	Steady

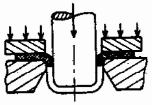
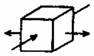
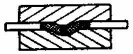
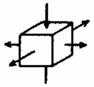
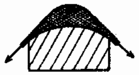
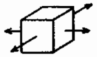
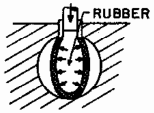
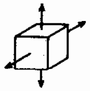
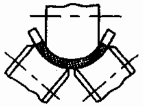
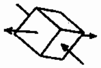
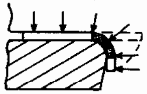
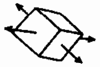
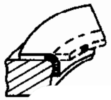
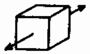
Basic Deformation Mechanism	No.	Forming Process: Name	Forming Process: Diagram	Approximate State of Stress at Square	Instantaneous Zone of Deformation (Cross-Hatched)	State of Deformation
Drawing (chipless forming) —*contd.*	12	Deep drawing of cups, boxes, and other shapes			Whole flange	Nonsteady
	13	Embossing (also beading and dimpling)			Zone in die cavity	Nonsteady
	14	Stretch forming			Whole part	Nonsteady
	15	Bulging			Zone in die cavity	Nonsteady
	16	Roll forming			Zone between rolls	Steady
Bending (chipless forming)	17	Straight flanging			In bend and in flange	Nonsteady
	18	Stretch flanging (concave flanges)			In bend and in flange	Nonsteady

TABLE 1.1—contd.

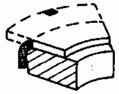
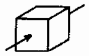
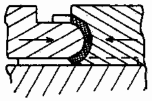
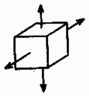
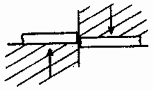
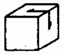
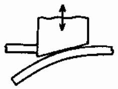
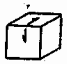
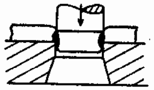
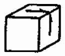
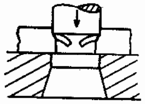
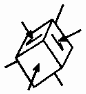
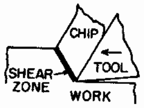
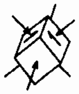
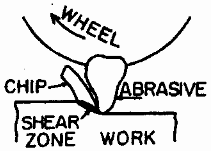
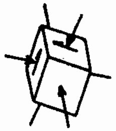
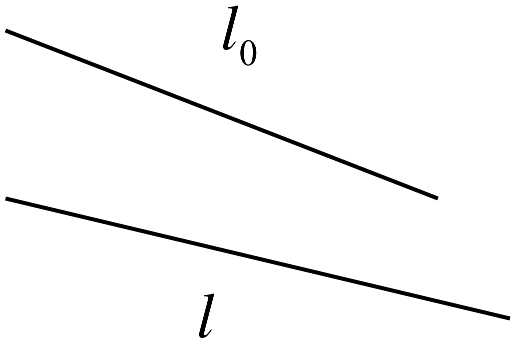
Basic Deformation Mechanism	No.	Forming Process: Name	Forming Process: Diagram	Approximate State of Stress at Square	Instantaneous Zone of Deformation (Cross-Hatched)	State of Deformation
Bending (chipless forming)—*contd.*	19	Shrink flanging (convex flanges)			In bend and in flange	Nonsteady
	20	Seaming			In bend	Nonsteady
Cutting and shearing (chipless forming)	21	Press shearing and cutting off			In narrow shear zone	Nonsteady
	22	Notching and nibbling			Under tool	Nonsteady
	23	Piercing and blanking			In narrow shear zone	Nonsteady
Cutting (chip forming)	24	Shaving			In narrow shear zone	Steady
	25	Machining as in turning, drilling, boring, milling, broaching, sawing			In narrow shear zone	Steady, except in some milling operations

TABLE 1.1—*contd.*

Basic Deformation Mechanism	*No.*	*Forming Process*		*Approximate State of Stress at Square*	*Instantaneous Zone of Deformation (Cross-Hatched)*	*State of Deformation*
		Name	*Diagram*			
Cutting (chip forming) —*contd.*	26	Grinding (also in lapping, honing, polishing)			In narrow shear zone	Quasisteady

This table is based on a modified classification due to Prof. Dr. Ing. O. Kienzle of the Tech. Univ. of Hannover, Germany. The instantaneous deformation zone is shown shaded and the state of stress is given for the element indicated in the deformation zone.

Strain (Lecture 2)



Engineering Strain, $e = \frac{l - l_0}{l_0} = \frac{\Delta l}{l_0}$ (1-18)

True Incremental strain, $d\varepsilon = \frac{dl}{l}$ (1-19)

Incremental strain, $\mathrm{d} \in = \frac{dl}{l} = -\frac{\mathrm{dA}}{\mathrm{A}}$

Total true strain,

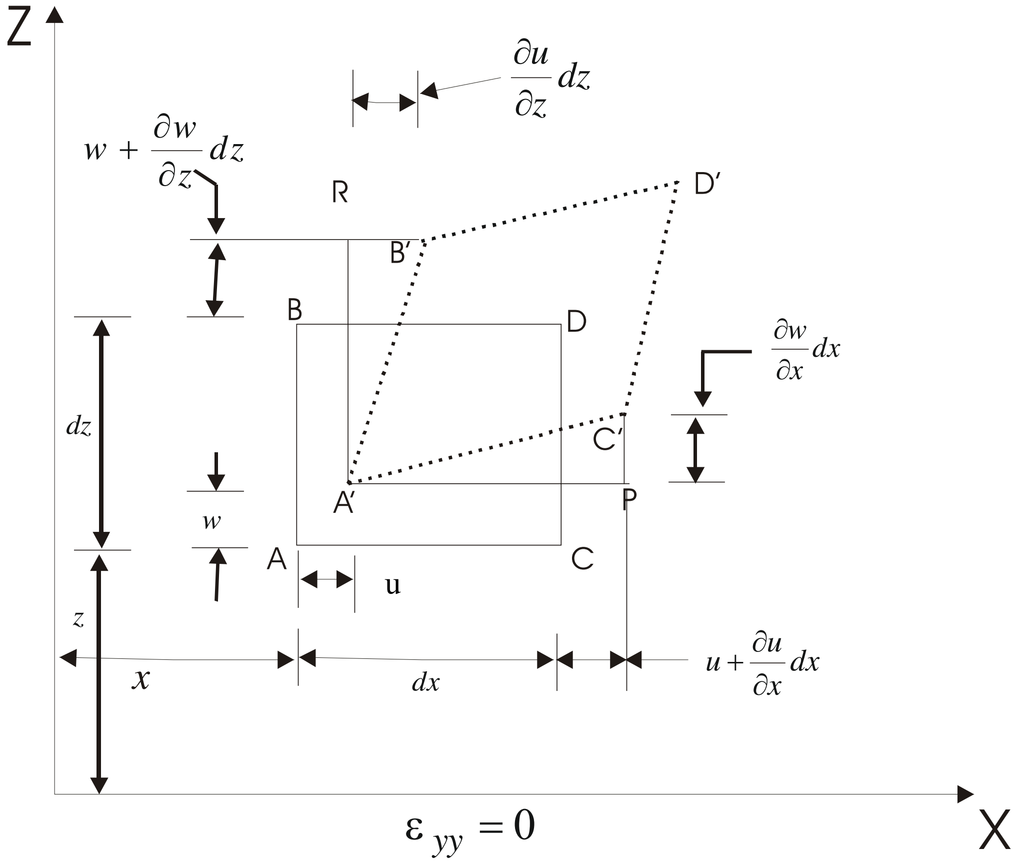
$$\in = \int \mathrm{d} \in = \int_{\ell_o}^{\ell} \frac{\mathrm{d}\ell}{\ell} = [\ln \ell]_{\ell_o}^{\ell} = \ln \frac{\ell}{\ell_o}$$

Two-Dimensional or Plane Strain Deformation

$ABCD$: small undeformed rectangular element

$A'B'C'D'$: after deformation (parallelogram)

u, w: displacements along X and Z respectively


Z
$\frac{\partial u}{\partial z} dz$
$w + \frac{\partial w}{\partial z} dz$
R
D′
B′
B
D
$\frac{\partial w}{\partial x} dx$
dz
C′
A′
P
w
A
C
u
z
x
dx
$u + \frac{\partial u}{\partial x} dx$
X

$$\varepsilon_{yy} = 0$$

Normal strains = extension/contraction

$$e_{xx} \overset{\text{def}^{\text{n}}}{=} \frac{\text{A'C'}-\text{AC}}{\text{AC}} = \frac{\text{A'C'}}{\text{AC}} - 1 \qquad (1\text{-}24)$$

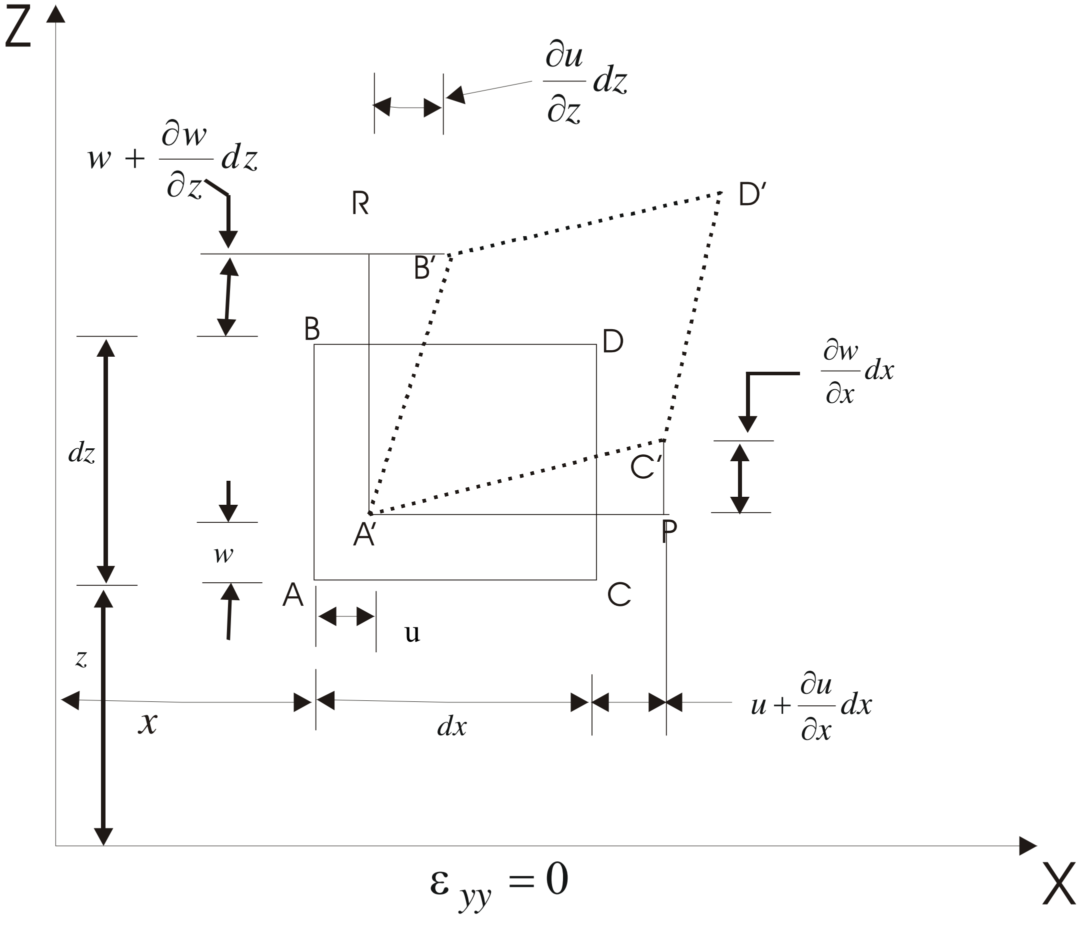
For small deformation:

$$A'P \approx A'C' \text{ and } \tan PA'C' \approx \angle PA'C'$$

$$e_{xx} \approx \frac{A'P}{AC} - 1 = \frac{dx - u + u + \frac{\partial u}{\partial x}dx}{dx} - 1 \qquad (1\text{-}25)$$

or

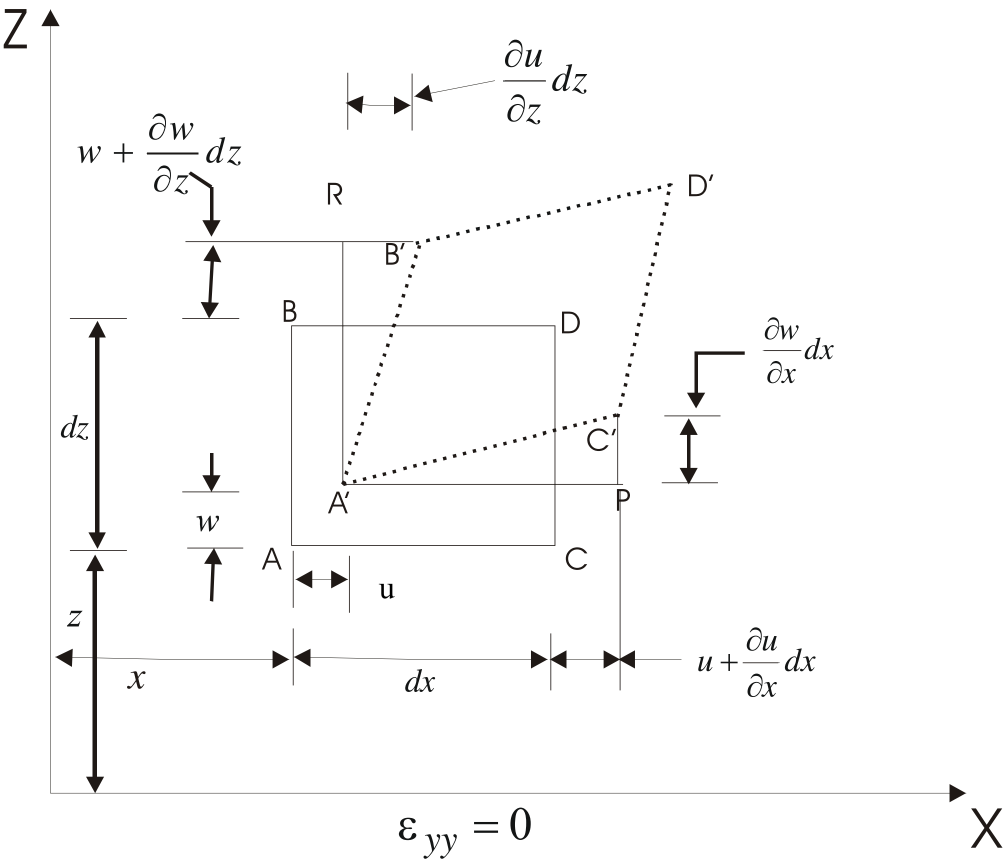
$$e_{xx} = \frac{\partial u}{\partial x} \qquad (1\text{-}26)$$


Z
$\frac{\partial u}{\partial z}dz$
$w + \frac{\partial w}{\partial z}dz$
R
D′
B′
B
D
$\frac{\partial w}{\partial x}dx$
dz
C′
A′
P
w
A
C
u
z
x
dx
$u + \frac{\partial u}{\partial x}dx$
$\varepsilon_{yy} = 0$
X

Similarly, $$e_{zz} = \frac{\partial w}{\partial z}$$

Shear strains =Angular distortions ($\angle$s RA'B' & PA'C')

$$\angle PA'C' \approx \frac{\partial w}{\partial x} \quad \text{and} \quad \angle RA'B' \approx \frac{\partial u}{\partial z}$$


Z
$\frac{\partial u}{\partial z} dz$
$w + \frac{\partial w}{\partial z} dz$
R
D′
B′
B
D
$\frac{\partial w}{\partial x} dx$
dz
C′
A′
P
w
A
C
u
z
x
dx
$u + \frac{\partial u}{\partial x} dx$
$\varepsilon_{yy} = 0$
X

Total shear strain, $$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \qquad (1\text{-}29)$$

3-D:

Normal Strains: $$e_{xx} = \frac{\partial u}{\partial x}$$

$$e_{yy} = \frac{\partial v}{\partial y}$$

$$e_{zz} = \frac{\partial w}{\partial z}$$

Shear Strains:

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

(where v is the displacement in the y direction)

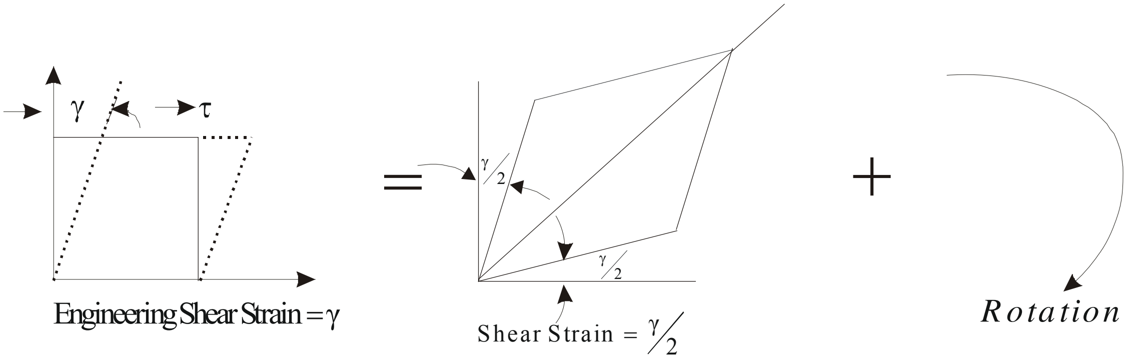
Strain tensor

$$e_{ij} = \begin{vmatrix} e_{xx} & e_{yx} & e_{zx} \\ e_{xy} & e_{yy} & e_{zy} \\ e_{xz} & e_{yz} & e_{zz} \end{vmatrix} \qquad (1\text{-}31)$$

Small deformations (occurs in pure shear): e_{ij} is a tensor

Large deformation (involves "simple" or engineering shear, simple shear = pure shear + rotation): e_{ij} is not a tensor

Metal forming involves large deformation (or simple shear)



(or simple shear) (pure shear) (rotation)

Tensor shear strain, $e_{xy} = \frac{1}{2}\gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$ (1-32)

Tensor (or pure) shear strain in half the engineering shear strain.

Notation: $e_x = e_{xx}$ (normal) and $\frac{1}{2}\gamma_{xy} = e_{xy}$ (shear)

Strain Transformation Equations:

Normal strain, e_x, transforms the same way as σ_x

Pure shear, $\gamma_{xy}/2$, transforms the same way as τ_{xy}

Plane strain:

$$e_z = \gamma_{xz} = \gamma_{zy} = 0 \quad (e_x, e_y \text{ and } \gamma_{xy} \neq 0)$$

$$e_{ij} = \Sigma\Sigma\, \ell_{im}\, \ell_{jn}\, e_{mn} \qquad \text{(A)}$$

(A)$\Rightarrow$ $\quad e_{x'} = e_x l^2{}_{x'x} + e_y l^2{}_{x'y} + \gamma_{xy} l_{x'x} l_{x'y} \qquad$ (1-34)

(A)$\Rightarrow$ $\quad \gamma_{x'y'} = 2e_x l_{x'x} l_{y'x} + 2e_y l_{x'y} l_{y'y} + \gamma_{xy}(l_{x'x} l_{y'y} + l_{y'x} l_{x'y}) \qquad$ (1-35)

(1-34), (1-35) $\Rightarrow$ strains transformation equations:

$$e_{x'} = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$e_{y'} = \frac{e_x + e_y}{2} - \frac{e_x - e_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\gamma_{x'y'} = -(e_x - e_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

Principal normal strains:

$$e_1, e_2 = \frac{e_x + e_y}{2} \pm \frac{1}{2}[(e_x - e_y)^2 + \gamma^2{}_{xy}]^{1/2} \quad (1\text{-}36)$$

Maximum engineering shear strain,

$$\gamma_{max} = [(e_x - e_y)^2 + \gamma^2{}_{xy}]^{1/2} \quad (1\text{-}37)$$

Strains on a plane (in terms of principal strains):

$$e_x, e_y = \frac{1}{2}(e_1 + e_2) \pm \frac{1}{2}(e_1 - e_2)\cos 2\theta \quad (1\text{-}38)$$

$$\gamma_{xy} = (e_1 - e_2)\sin 2\theta \quad (1\text{-}39)$$

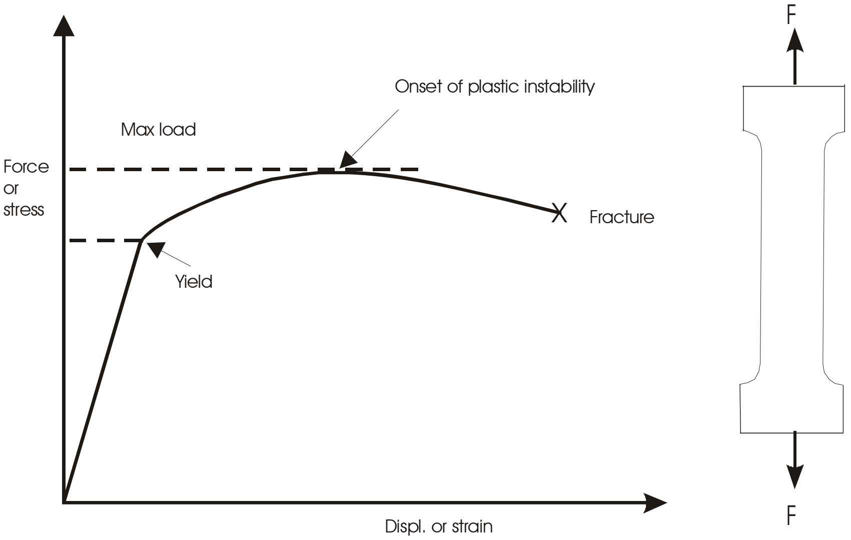
Mohr's circle for strain is identical to Mohr's circle for stress.

Simply, replace τ by $\gamma/2$ along the ordinate.

Plastic Instability (Lecture 8, Chapter 4)

Plastic instability refers to deformation under falling load, pressure, stress etc. - deformation process is unstable - deformation starts to localize into a small region of material.

Instability under uniaxial tension:



$$\overline{\sigma} = (F/A) \quad \text{or} \quad F = \overline{\sigma} A \tag{4-1}$$

At max load, dF=0 $\Rightarrow \mathrm{d}F = \overline{\sigma}\mathrm{d}A + A\mathrm{d}\overline{\sigma} = 0$ (4-2)

$$(4\text{-}2) \Rightarrow \frac{\mathrm{d}\overline{\sigma}}{\overline{\sigma}} = -\frac{\mathrm{d}A}{A} = \mathrm{d}\overline{\varepsilon} \quad \Rightarrow \frac{\mathrm{d}\overline{\sigma}}{\mathrm{d}\overline{\varepsilon}} = \overline{\sigma} \tag{4-4}$$

Also, $\overline{\sigma} = K\overline{\varepsilon}^{n} \Rightarrow \frac{\mathrm{d}\overline{\sigma}}{\mathrm{d}\overline{\varepsilon}} = Kn\overline{\varepsilon}^{n-1} = \frac{n}{\varepsilon} K\overline{\varepsilon}^{n}$ (4-5)

$$(4-4),(4-5) \Rightarrow \quad \overline{\sigma} = \frac{n}{\overline{\varepsilon}} K\overline{\varepsilon}^{n} = \frac{n}{\overline{\varepsilon}}\overline{\sigma}$$

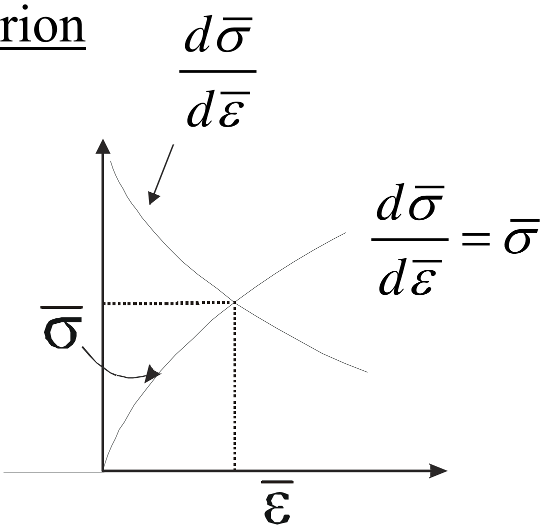
$$\therefore (n/\overline{\varepsilon}) = 1 \text{ or } \overline{\varepsilon} = n \tag{4-6}$$

Plastic instability under uniaxial tension (cont'd.)

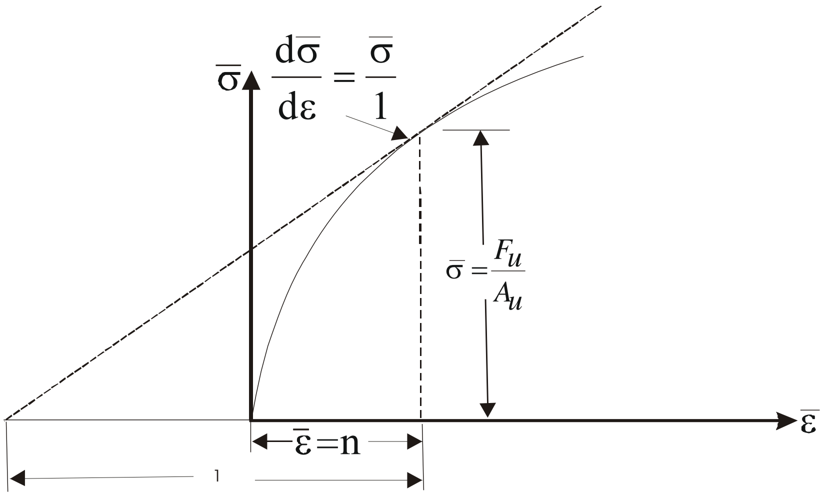
When $\bar{\varepsilon} \rightarrow n \Rightarrow F \rightarrow F_{max}, \sigma \rightarrow \sigma_{UTS}$,

$\Rightarrow$ material reaches its instability point.

Considere criterion



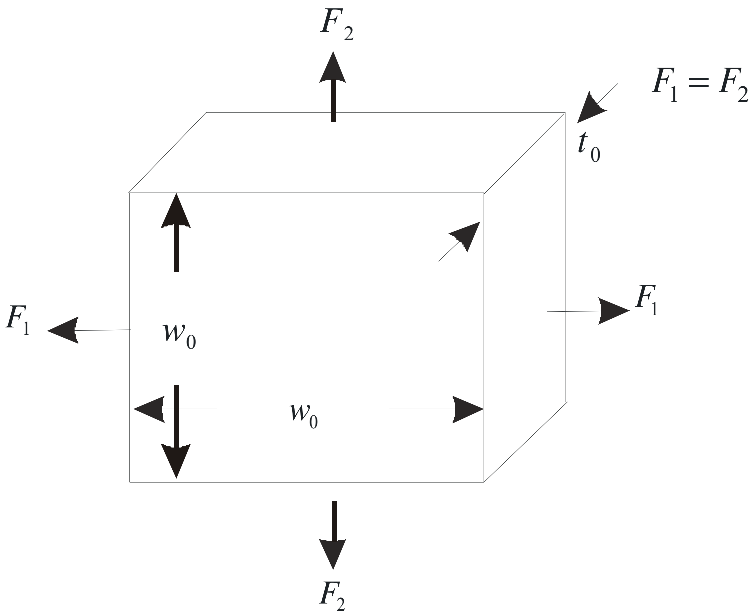
Graphical construction



At instability point, the magnitude of slope, $\frac{d\bar{\sigma}}{d\bar{\varepsilon}}$, is equal to σ.

Plastic instability under balanced biaxial tension

For example, consider balanced biaxial stretching of a thin sheet $(F_1 = F_2)$:



$$F_1 = \sigma_1 A_1 \qquad \text{(A)}$$

so $$dF_1 \overset{(A)}{=} 0 = \sigma_1 dA_1 + A_1 d\sigma_1 \qquad \text{(4-7)}$$

or $$\frac{d\sigma_1}{\sigma_1} = -\frac{dA_1}{A_1} = d\varepsilon_1 \qquad \text{(4-8)}$$

$$\overline{\sigma} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

Balanced biaxial stress state: $\sigma_1 = \sigma_2,\ \sigma_3 = 0$

$$\therefore \overline{\sigma} = \frac{1}{\sqrt{2}}[\sigma_1{}^2 + \sigma_1{}^2]^{1/2} = \sigma_1 (= \sigma_2) \qquad \text{(4-9)}$$

Plastic instability under balanced biaxial tension

Effective Strain: $\bar{\varepsilon} = \left[\frac{2}{3}(\varepsilon_1{}^2 + \varepsilon_2{}^2 + \varepsilon_3{}^2)\right]^{1/2}$ (2-21)

Balanced biaxial strain state: $\varepsilon_1 = \varepsilon_2$

Incompressibility $\Rightarrow \varepsilon_3 = -(\varepsilon_1 + \varepsilon_2) = -2\varepsilon_1$

$$\bar{\varepsilon} = \left[\frac{2}{3}(\varepsilon_1{}^2 + \varepsilon_1{}^2 + 4\varepsilon_1{}^2)\right]^{1/2} = 2\varepsilon_1 = 2\varepsilon_2 = -\varepsilon_3 \quad (4\text{-}10)$$

$(4-9),(4-10) \rightarrow (4-8) \Rightarrow$

$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{d\bar{\sigma}}{d\sigma_1} \times \frac{d\sigma_1}{d\bar{\varepsilon}} = \frac{d\bar{\sigma}}{d\sigma_1} \times \frac{d\sigma_1}{d\varepsilon_1} \times \frac{d\varepsilon_1}{d\bar{\varepsilon}}$$

or $$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = (1) \times (\sigma_1) \times \left(\frac{1}{2}\right)$$

or $$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{\bar{\sigma}}{2} \quad (4\text{-}11)$$

Also $$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = nK\bar{\varepsilon}^{n-1} = \frac{n}{\bar{\varepsilon}}\bar{\sigma} \quad (B)$$

(4-11) and (B) $\Rightarrow$ $\bar{\varepsilon} = 2n$ (4-12)

∴ Biaxial instability strain is twice the uniaxial instability strain.

Thin walled sphere under internal pressure

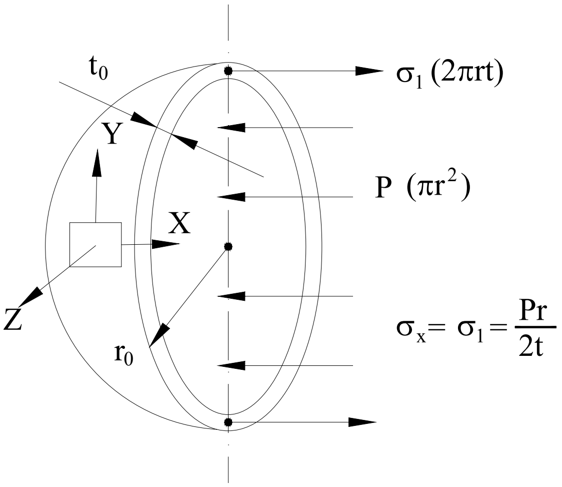
Consider free body diagram (FBD) of ½ of a spherical pressure vessel, as shown below:

Co-ordinate system:

$$1,\ 2,\ 3\ \Rightarrow\ X(axial),\ Y(hoop),\ Z\ (radial\ or\ thickness)$$

Symmetry $\Rightarrow \sigma_1 = \sigma_2\ (\sigma_3 = 0)$ (A)

Force balance in X (or 1) direction:



$$\sigma_1(2\pi rt) = P(\pi r^2)$$

or $$\sigma_1 = \frac{P(\pi r^2)}{2\pi rt} = \frac{\text{Pr}}{2t} \Rightarrow P = \frac{2\sigma_1 t}{r} \qquad (4\text{-}13)$$

where P = internal pressure
r, t = instantaneous radius and thickness
(at pressure P)

At instability, $P = P_{max}$ or $dP = 0$

Sphere under internal pressure (continued)

$$P \overset{(4-13)}{=} \frac{2\sigma_1 t}{r} \Rightarrow dP = 0 = \frac{2td\sigma_1}{r} + \frac{2\sigma_1 dt}{r} - \frac{2t\sigma_1 dr}{r^2} \qquad (4\text{-}14)$$

or
$$\frac{d\sigma_1}{\sigma_1} = \frac{dr}{r} - \frac{dt}{t} \qquad (4\text{-}15)$$

$$\frac{dr}{r} = d\varepsilon_1, \ \frac{dt}{t} = d\varepsilon_3 \qquad (B, C)$$

and
$$d\varepsilon_1 = d\varepsilon_2 = -\frac{d\varepsilon_3}{2} \qquad (D)$$

$$(B), (C), (D), (4\text{-}15) \Rightarrow \frac{d\sigma_1}{\sigma_1} = -\frac{3}{2} d\varepsilon_3 \qquad (4\text{-}16)$$

From definitions of $\bar{\sigma}$ and $\bar{\varepsilon}$, (A) and (D):

$$d\bar{\varepsilon} = -d\varepsilon_3, \bar{\sigma} = \sigma_1 \qquad (E)$$

$$(4-16), E \Rightarrow \frac{d\bar{\sigma}}{\bar{\sigma}} = \frac{3}{2} d\bar{\varepsilon} \quad \text{or} \quad \frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{3}{2}\bar{\sigma} \qquad (4\text{-}17)$$

Power law: $\bar{\sigma} = K\bar{\varepsilon}^n \Rightarrow \frac{d\bar{\sigma}}{d\bar{\varepsilon}} = nK\bar{\varepsilon}^{n-1} = \frac{n}{\bar{\varepsilon}}\bar{\sigma} \qquad (F)$

$$(4-17), (F) \Rightarrow \qquad \bar{\varepsilon} = (2/3)n \qquad (4\text{-}18)$$

Effective instability strain under internal pressure is 2/3rd of instability strain in uniaxial tension.

Sphere under internal pressure (Continued)

Calculation of pressure:

$$\varepsilon_3 = \ell n\left(\frac{t}{t_0}\right) \Rightarrow t = t_0 \exp(\varepsilon_3) = t_0 \exp(-\bar{\varepsilon}) \qquad (4\text{-}19)$$

$$\varepsilon_1 = \ell n\left(\frac{r}{r_0}\right) \Rightarrow r = r_0 \exp(\varepsilon_1) = r_0 \exp(\bar{\varepsilon}/2) \qquad (4\text{-}20)$$

$$\sigma_1 = \bar{\sigma} \overset{(4-13)}{=} \frac{Pr}{2t} \underset{(4-19)}{\overset{(4-20)}{=}} \frac{Pr_0 \exp(\bar{\varepsilon}/2)}{2t_0 \exp(-\bar{\varepsilon})}$$

or $$P = 2\bar{\sigma}\left(\frac{t_0}{r_0}\right) \exp\left(-\frac{3}{2}\bar{\varepsilon}\right)$$

or $$P = 2K\bar{\varepsilon}^n \left(\frac{t_0}{r_0}\right) \exp\left(-\frac{3}{2}\bar{\varepsilon}\right) \qquad (4\text{-}21)$$

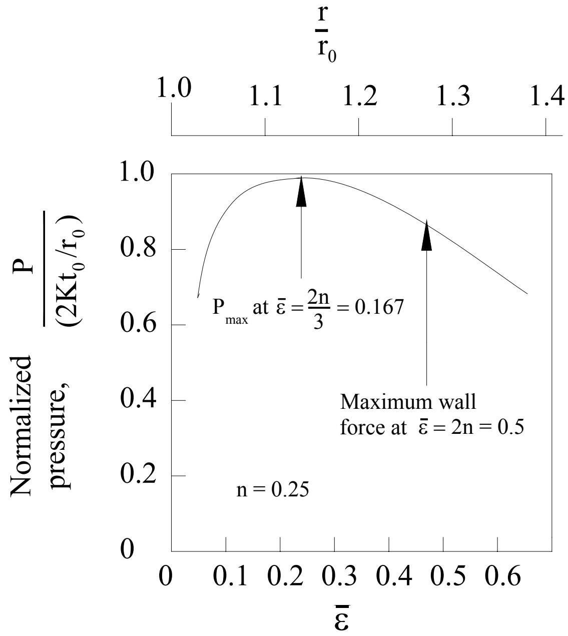
Since $P = P_{\max}$ when $\bar{\varepsilon} \overset{(4-18)}{=} \frac{2}{3}n$, one obtains:

$$P_{\max} \overset{(4-21)}{=} 2K\left(\frac{t_0}{r_0}\right)\left(\frac{2}{3}n\right)^n \exp(-n) \qquad (4\text{-}22)$$

Sphere under internal pressure (Continued)

$$P = 2K\bar{\varepsilon}^{n}\left(\frac{t_0}{r_0}\right)\exp\left(-\frac{3}{2}\bar{\varepsilon}\right) \qquad (4\text{-}21)$$

A plot of equation (4-21) gives:



Remarks:

[1]. Max pressure occurs at $\bar{\varepsilon} = (2/3)n$ (instability). However, deformation continues and no strain localization or failure results.

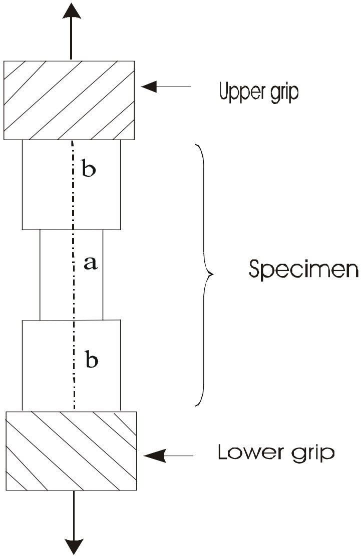
[2]. Force in the walls continues to increase until $\bar{\varepsilon} = 2n$. Even at this point, localized deformation can not result in a sharp neck because:

sharp neck $\Rightarrow$ small r $\overset{(4\text{–}13)}{\Rightarrow}$ small $\bar{\sigma}$ in the neck.

Therefore, any localized failure will be triggered by a defect or inhomogeneity.

Effect of inhomogeneity on uniform strain

Consider a tensile specimen with a geometric inhomogeneity at the centre – a slightly smaller dimension in region a compared to b, as shown:



Define an inhomogeneity factor, $f \overset{def^n}{=} \frac{A_{a0}}{A_{b0}}$ (4-23)

Since $A_{a0} < A_{b0}, \; f < 1$

(lower $f \Rightarrow$ stronger 'inhomogeneity' and vice versa)

Force balance: $F_a = F_b \Rightarrow A_a\sigma_a = A_b\sigma_b$ (4-24)

Assuming power law, $\sigma = K\varepsilon^n, (4-24) \Rightarrow$

$$A_{a0}\exp(-\varepsilon_a)K\varepsilon_a^n = A_{b0}\exp(-\varepsilon_b)K\varepsilon_b^n \qquad (4\text{-}25)$$

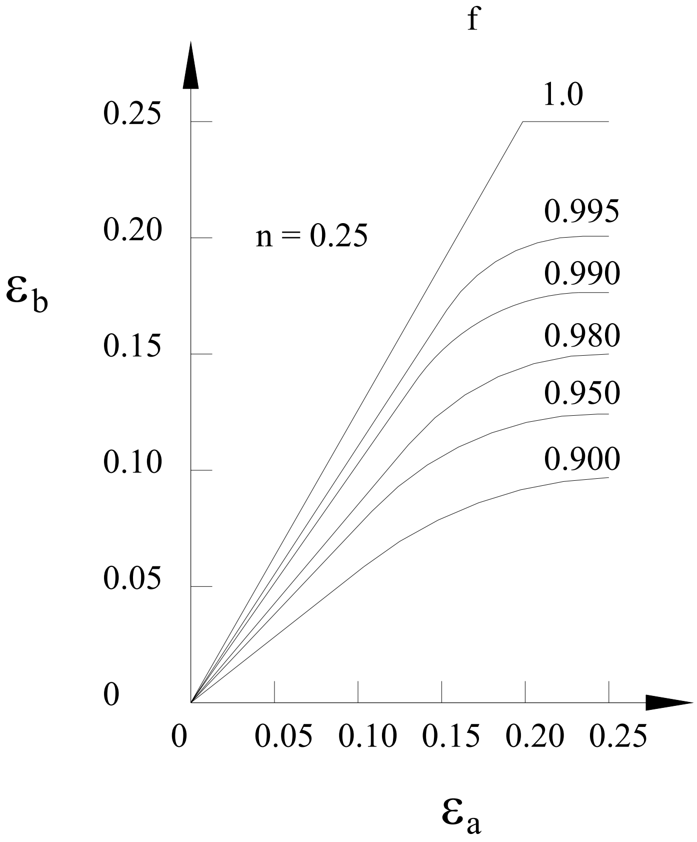
$$(4-23) \rightarrow (4-25) \Rightarrow f\varepsilon_a^n\exp(-\varepsilon_a) = \varepsilon_b^n\exp(-\varepsilon_b) \qquad (4\text{-}26)$$

Numerical solution of equation 4-26 for different f values (assuming n=0.25)

$$f\varepsilon_a^n \exp(-\varepsilon_a) = \varepsilon_b^n \exp(-\varepsilon_b) \qquad (4\text{-}26)$$

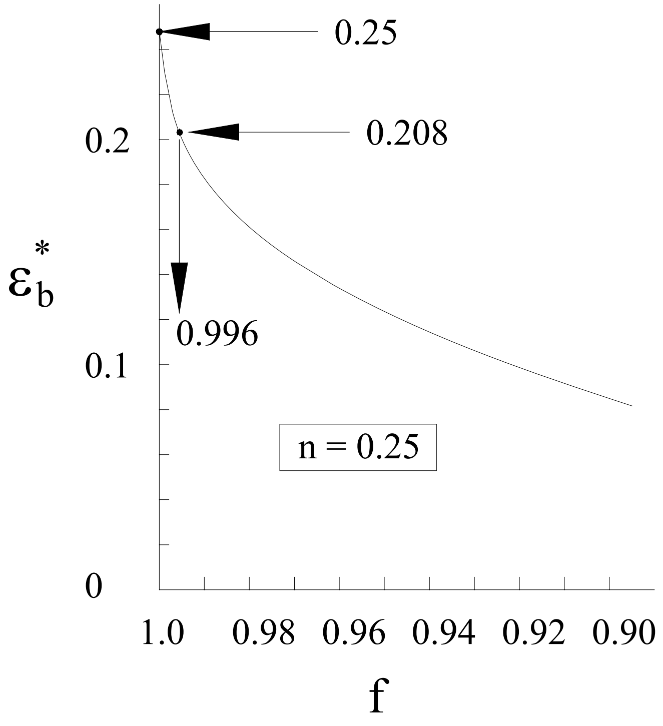
Assume f and n, then calculate ε_a and ε_b.

$$\varepsilon_a \to \infty \Rightarrow \varepsilon_b \to \varepsilon_b^* (\underline{\text{limiting}} \text{ value})$$



Numerical solution of equation 4-26 for different f values (assuming n=0.25)

$$f\varepsilon_a^n \exp(-\varepsilon_a) = \varepsilon_b^n \exp(-\varepsilon_b) \qquad (4\text{-}26)$$



Note:

Even a small inhomogeneity has an appreciable effect. For example, consider a tensile specimen of 0.505 in diameter and maximum variation of 0.001.

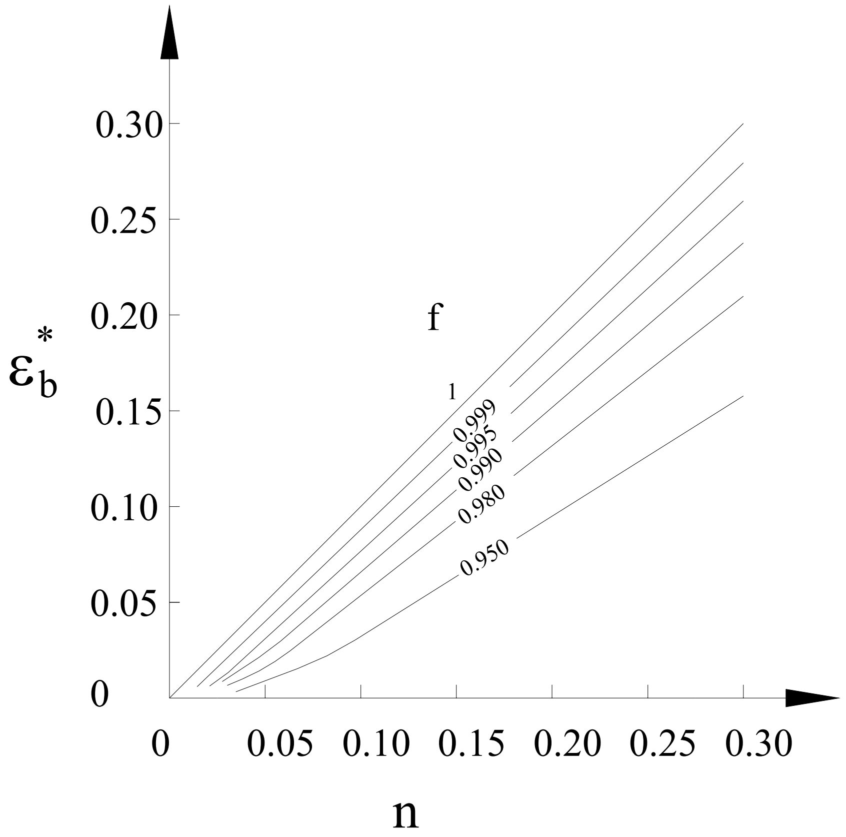
$$f = \frac{A_{a0}}{A_{b0}} = \frac{(d_{a0})^2}{(d_{bo})^2} = \frac{(0.505)^2}{(0.504)^2} = 0.996 \Rightarrow \varepsilon_b^* = 0.208$$

Now compare this value to $\varepsilon_b^* = 0.25$ (for f = 1)!

Numerical solution of equation 4-26 (continued)

Effect of n-value:

$$f\varepsilon_a^n \exp(-\varepsilon_a) = \varepsilon_b^n \exp(-\varepsilon_b) \qquad (4\text{-}26)$$



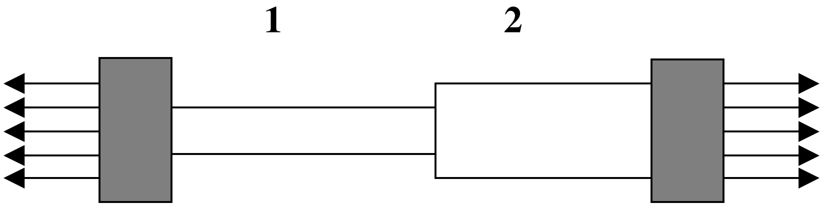
Example Problem (prob. # 4-12 from text)

Consider a material which undergoes linear strain hardening. Its true stress-strain curve in tension is given by $\sigma = Y + 1.35Y\varepsilon$.

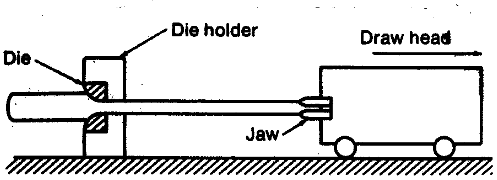
[1]. At what value of true strain will necking start?

[2]. Suppose a stepped tensile bar is made from this material. The initial cross-sectional area of region 1 is 0.990 times the initial cross-sectional area of region 2.

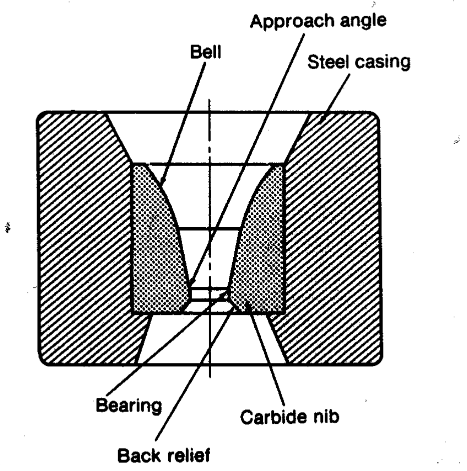
What is the strain in region 2 when the strain in region 1 reaches 0.200?

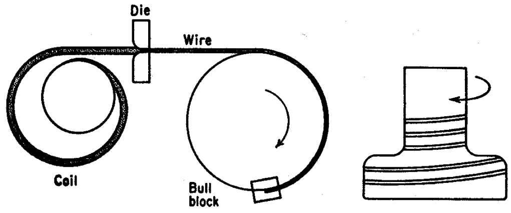


Wire or Rod Drawing
(Chapter 7, Lecture 13)



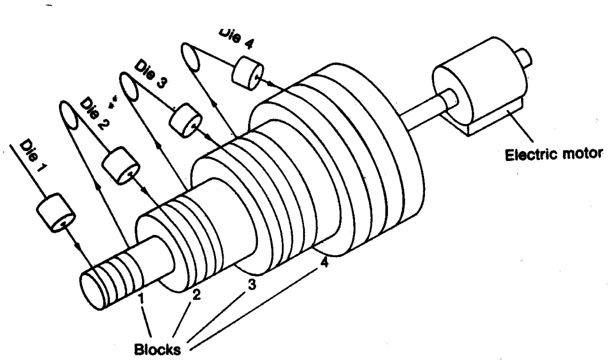
(a)



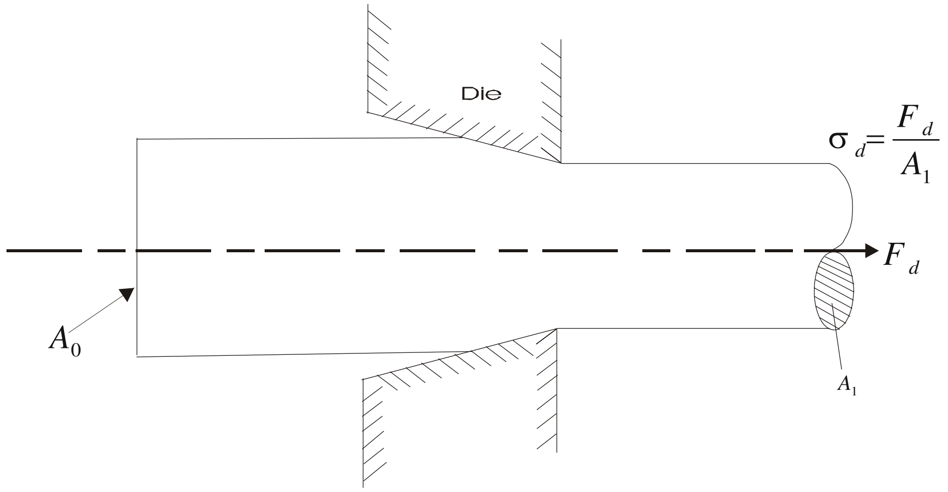


Top view

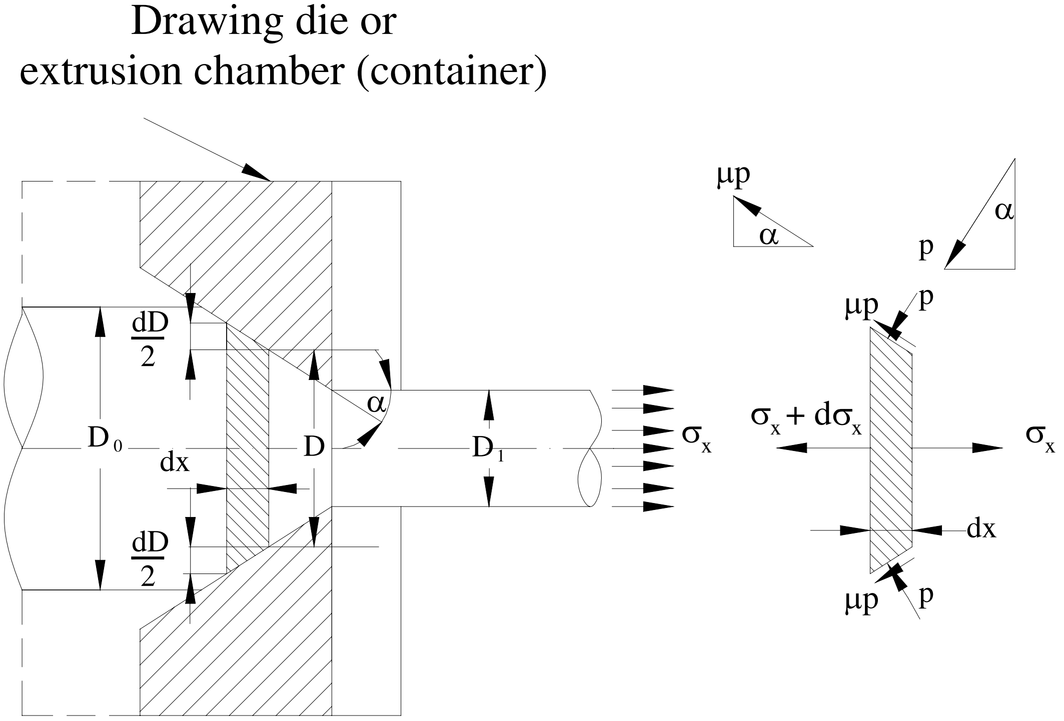
Side view of bull block



Analysis of wire drawing process by slab method:

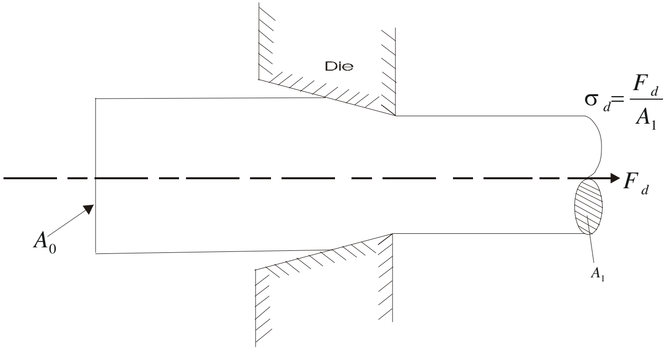


Consider forces acting on a conical element:



Basic governing equations are similar to strip drawing (equation 7-8 earlier) as well as extrusion.

Analysis of wire or rod drawing (Continued)



Expression for drawing stress:

$$\sigma_d = \bar{\sigma}\left(\frac{1+B}{B}\right)\left[1-\exp\left(-B\varepsilon_h\right)\right] \quad (7\text{-}14)$$

$$\bar{\sigma} \approx Y_m \left(for\ \varepsilon_h \geq \varepsilon \geq 0\right),\ \ Y_m = \text{mean flow stress}$$

where $B = \mu \cot\alpha$

$$\varepsilon_h = 2\ln\left(\frac{d_0}{d_1}\right)$$

Assumptions used in the derivation:

α is small
redundant strain is negligible
reduction is low
μ is low (sliding friction).

Assuming sticking friction, $\mu P = k$:

$$\sigma_d = \left(1 + 1/\sin 2\alpha\right)\cdot 2k\cdot\varepsilon_h$$

Example – Wire drawing (Problem 7-10 of text)

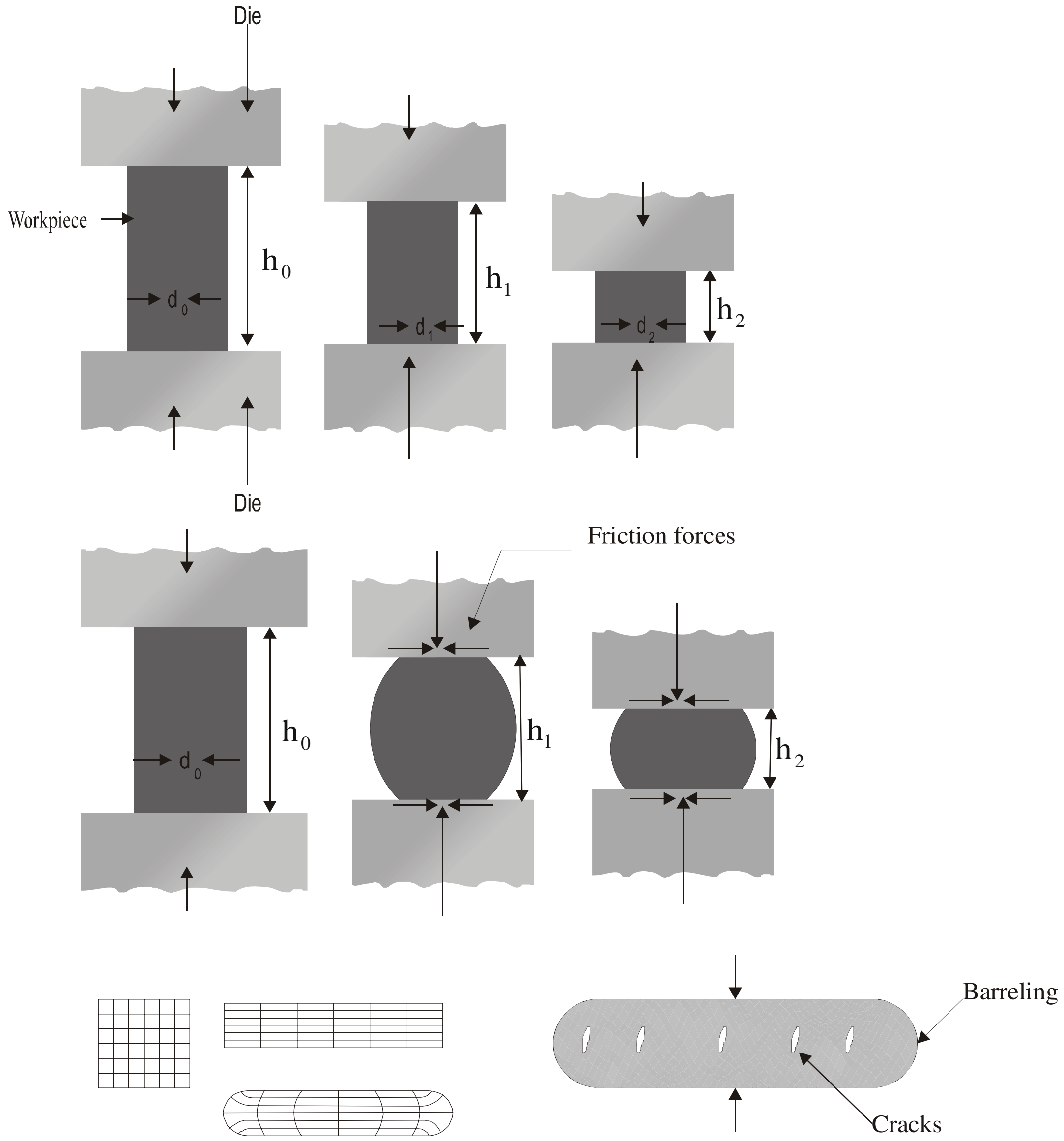
A metal having a constant flow stress of 5,000 psi is to be drawn from a 4-in diameter to a 2-in diameter using a die of 30° semi-angle.

Calculate the drawing stress if the friction condition at the work piece interface is:

[1]. Frictionless
[2]. Sticking
[3]. $\mu = 0.20$

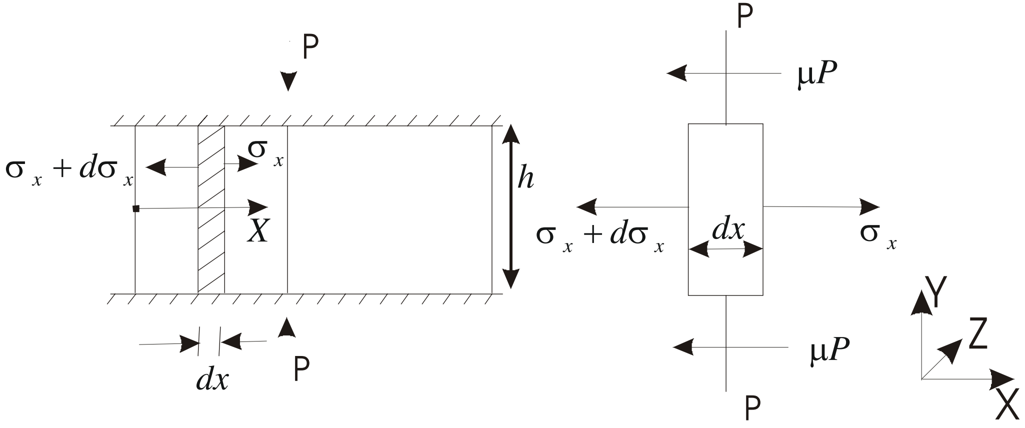
Open Die Forging

(Chapter 7, Lecture 14)



Direct compression in plane strain of a block - slab analysis

Pressure is applied normal to the surface of the block



Expression for pressure distribution:

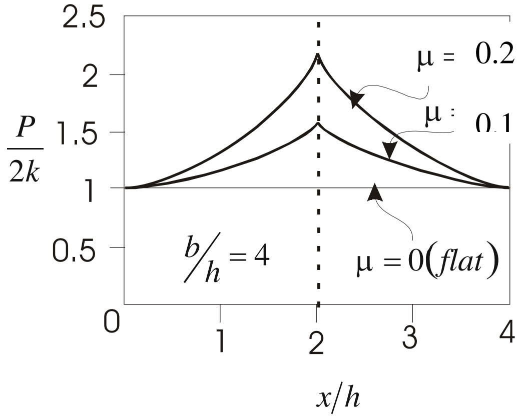
or $$\frac{P}{2k} = \exp\frac{2\mu x}{h} \qquad (7\text{-}18)$$

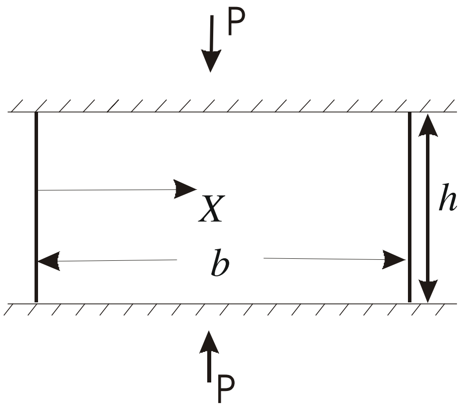
Eqn. (7-18) gives pressure as a function of x:

(for a given block height h, material shear strength k, and coefficient of friction μ).

Rise of P towards centreline ("friction hill"):

$$\frac{P}{2k} = \exp\frac{2\mu x}{h} \qquad (7\text{-}18)$$





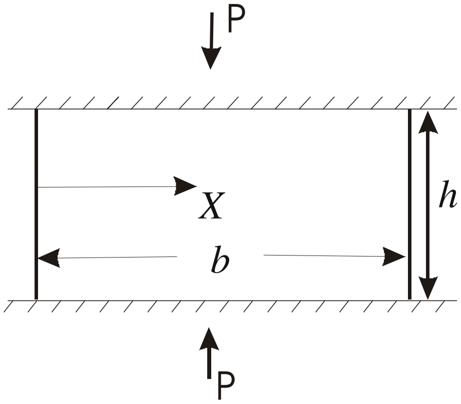
Maximum pressure, $P_{\max}$, occurs at the centre line where $x = b/2$

$$\therefore \left(\frac{P}{2k}\right)_{\max} \overset{(7-18)}{=} \exp\left(\frac{\mu b}{h}\right) \qquad (7\text{-}19)$$

Note: This is a sliding friction case.

Average pressure, P_{av} (for **sliding** friction case):

$$\frac{P}{2k} = \exp\frac{2\mu x}{h} \qquad (7\text{-}18)$$



First, calculate average force, F_y (in Y direction):

$$F_y = \int_0^{b/2} P\,dx \overset{(7-18)}{=} \int_0^{b/2} 2k\exp\left(\frac{2\mu x}{h}\right)dx$$

or

$$F_y = 2k\left(\frac{h}{2\mu}\right)\left[\exp\left(\frac{\mu b}{h}\right)-1\right]$$

$$\therefore \quad P_{av} = \frac{F_y}{(b/2)} = \frac{2kh}{\mu b}\left[\exp\left(\frac{\mu b}{h}\right)-1\right]$$

$$\therefore \quad \frac{P_{av}}{2k} = \frac{h}{\mu b}\left[\exp\left(\frac{\mu b}{h}\right)-1\right] \qquad (7\text{-}22)$$

Expanding: $\exp\left(\frac{\mu b}{h}\right)-1=1+\left(\frac{\mu b}{h}\right)+\frac{\left(\frac{\mu b}{h}\right)^{2}}{2!}+....-1$

and after neglecting higher order terms, one obtains:

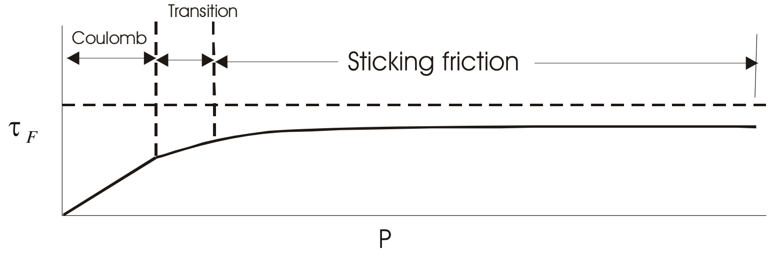
$$\frac{P_{av}}{2k}=1+\frac{1}{2}\frac{\mu b}{h} \qquad (7\text{-}23)$$

Average pressure, P_{av}, is a function of:

- geometry (b and h)
- friction (μ)
- flow stress of the material

[2]. Sticking friction at the interface $(\mu P = k)$

Pressure Distribution:



$$\frac{P}{2k} = 1 + \frac{x}{h}$$

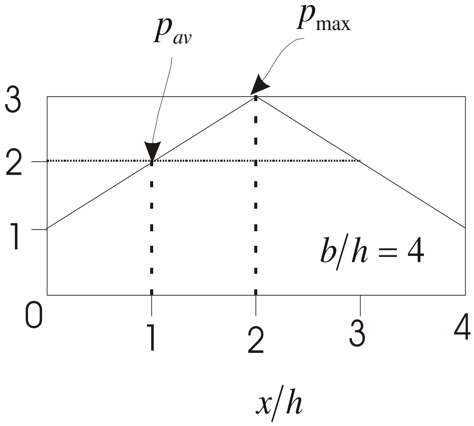
(Linear distribution of pressure with x)

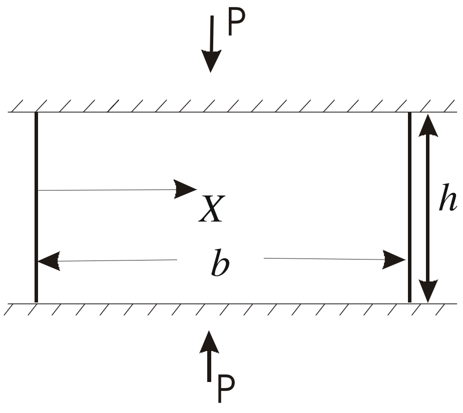
[1]. At the centre line (or mid-width)

$$\left(\frac{P}{2k}\right)_{\max} \overset{(x=b/2)}{=} 1+\frac{b}{2h}$$

[2]. At quarter-width

$$\frac{P_{av}}{2k} \overset{(x=b/4)}{=} 1+\frac{b}{4h}$$





Plane Strain Compression – Summary

Sliding friction $\mu P < k$	**Sticking Friction** $\mu P = k$
$P = 2k \exp \frac{2\mu x}{h}$ (7-18)	$P = 2k\left[1+\frac{x}{h}\right]$ (7-24)
$P_{\max} = 2k \exp \frac{\mu b}{h}$ (7-19)	$P_{\max} = 2k\left[1+\frac{b}{2h}\right]$ (7-25)
$P_{av} = \frac{2kh}{\mu b}\left[\exp\left(\frac{\mu b}{h}\right)-1\right]$ (7-22) $P_{av} = 2k\left[1+\frac{1}{2}\frac{\mu b}{h}\right]$ (7-23)	$P_{av} = 2k\left[1+\frac{b}{4h}\right]$ (7-26)

Example – Direct compression in plane strain

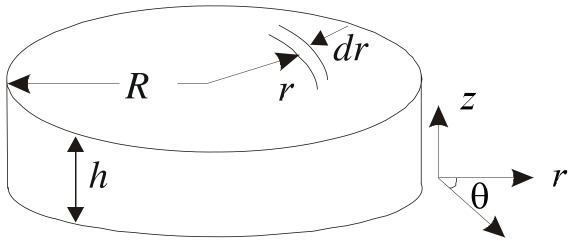
A rectangular work-piece has the following original dimensions: length (or b)=120 mm, height=30 mm, width = 20 mm. Material parameters of the work piece are given as: K=400 MPa and n=0.3. The work-piece is being forged in plane strain with $\mu = 0.2$. Calculate the force required for the height to be reduced by 20%.

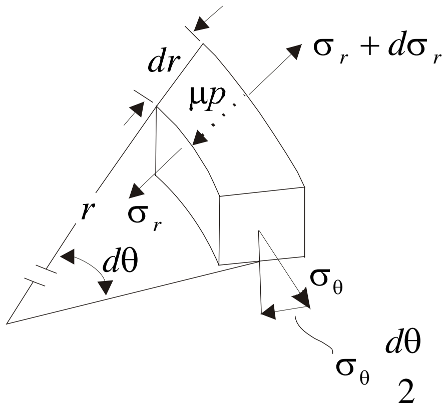
Axisymmetric compression

(Chapter 7, Lecture 15)

Slab analysis is similar to previous cases (strip drawing, axisymmetric wire drawing and plane strain compression of a block).

Element analyzed is a ring of differential thickness, dr, and height, h, and located at radius, r, from the centre.





Steps:

[1]. Force balance in the radial direction – gives a DE in terms of σ_r

[2]. Simplify DE with $\sigma_r = \sigma_\theta$

[3]. Simplify DE with yield criterion:

$$\sigma_r + P = Y, \ \ \sigma_Z = -P$$

[4]. Use B. C. to obtain a closed form expression for pressure distribution

Axisymmetric compression (Continued)

[1]. **Sliding friction:** $\mu P \le k \Rightarrow P \le k/\mu$

Pressure distribution:

$$P = Y \exp\left[\frac{2\mu}{h}(R-r)\right] \qquad (7\text{-}32)$$

$$(7\text{-}32) \Rightarrow \quad Y \exp\left[\frac{2\mu}{h}(R-r)\right] \le k/\mu$$

$\therefore$ Limiting radius, $r^* = R - \left(\frac{h}{2\mu}\right) \ell n \left(\frac{k}{\mu Y}\right)$

For sliding friction over the entire surface (r=0):

$$R \le \left(\frac{h}{2\mu}\right) \ell n \left(\frac{k}{\mu Y}\right)$$

Maximum pressure (r = 0):

$$P_{\max} \overset{(7-32)}{=} Y \exp\frac{2\mu R}{h}$$

Axisymm. Compression - Pressure Distribution

Average pressure P_a

$$P_a \overset{def^n}{=} \frac{1}{\pi R^2}\int_o^R P2\pi r dr \overset{(7-32)}{=} \frac{2Y}{R^2}\int_o^R r\exp\left[\frac{2\mu}{h}(R-r)\right]dr \quad (7\text{-}33)$$

or $$P_a = \frac{1}{2}\left(\frac{h}{\mu R}\right)^2 Y\left[\exp\left(\frac{2\mu R}{h}\right) - \left(\frac{2\mu R}{h}\right) - 1\right] \quad (7\text{-}34)$$

Approximation of (7-34):

$$P_a = Y\left(1 + \frac{a}{3} + \frac{a^2}{12} + \ldots\right) \text{ where } a = \frac{2\mu R}{h} \quad (7\text{-}35)$$

$$P_a = Y\left(1 + \frac{2\mu R}{3h}\right)$$

Axisymmetric Compression

[2]. Sticking friction: $\mu P = k$

Pressure distribution:

$$P = Y + (2k/h)(R - r) \qquad (7\text{-}37)$$

Maximum pressure:

(7-37) $\Rightarrow$ $P = Y + (2kR/h)$

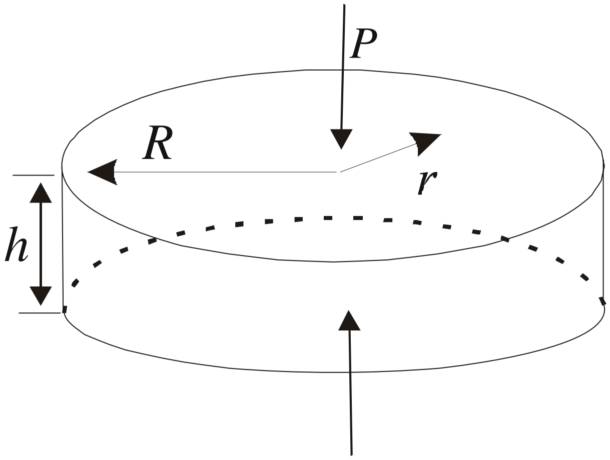
Average pressure:

$$P_a = \frac{1}{\pi R^2}\int_o^R P \cdot 2\pi r dr = \frac{1}{\pi R^2}\int_0^R [Y + \frac{2k}{h}(R - r)]2\pi r dr$$

$$P_a = Y + \frac{2kR}{3h} \qquad (7\text{-}38)$$

Axisymmetric Compression – Summary

Sliding friction $\mu P < k$	**Sticking Friction** $\mu P = k$
$P = Y \exp\left(\frac{2\mu}{h}(R-r)\right)$ (7-32)	$P = Y + \frac{2k}{h}(R-r)$ (7-37)
$P_{\max} = Y \exp\frac{2\mu R}{h}$	$P_{\max} = Y + \frac{2kR}{h}$
$P_{av} = Y\left(1 + \frac{a}{3} + \frac{a^2}{12} + \ldots\right)$ (7-35) where $a = \frac{2\mu R}{h}$	$P_{av} = Y + \frac{2kR}{3h}$ (7-38)



Example – Axisymmetric compression of a cylindrical work piece

Given:

SAE 1040 steel cylinder being forged at a high temperature. Yield stress at forging temperature = 6000 psi.

Starting size: 3 in. high and 1 in diameter
Finished size: 1.5” high

Calculate:

[a]. If $\mu = 0.4$, determine the average force required for upsetting.

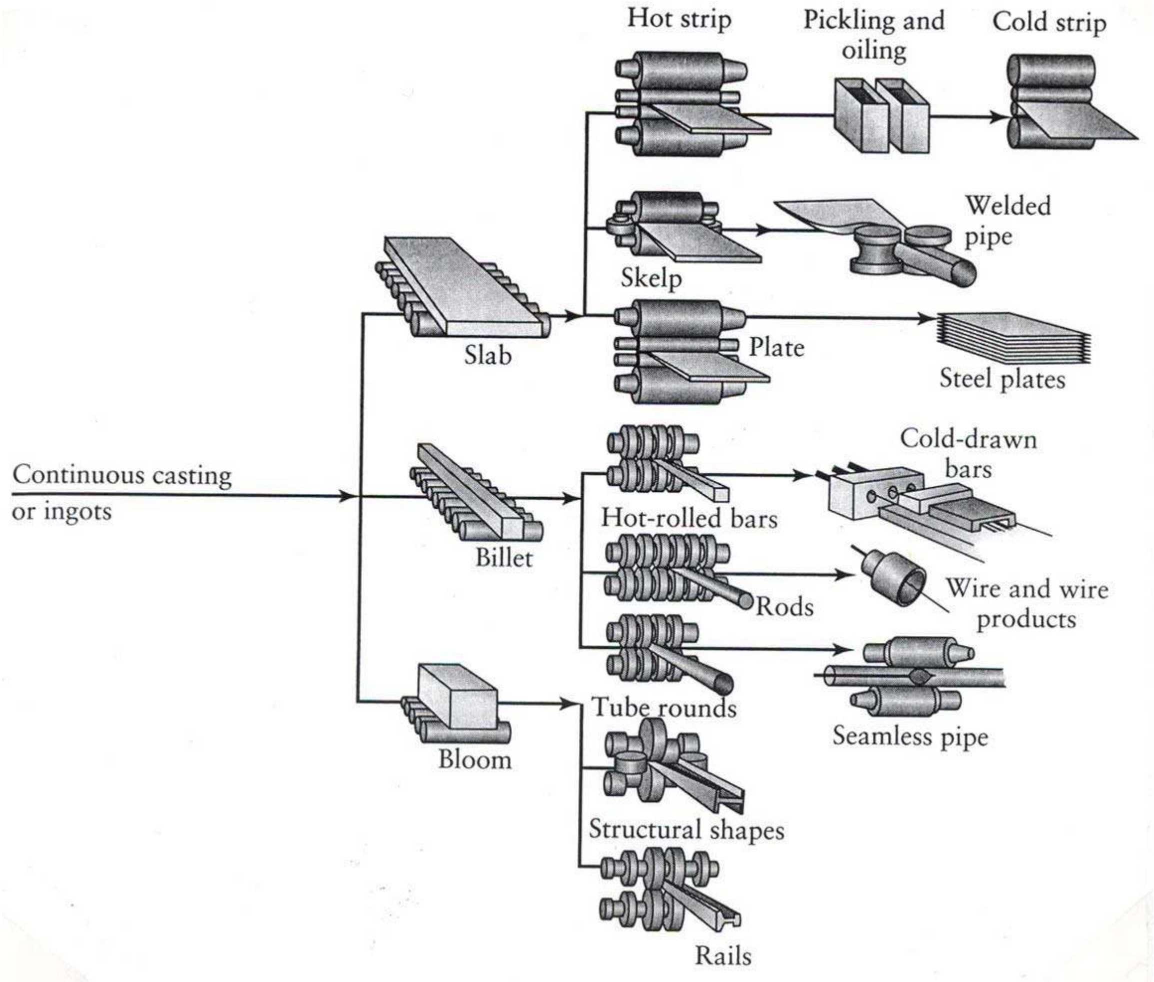
[b]. How much extra force is required over what would be needed if friction was absent.

[c]. If it takes 3 seconds to produce the forging and efficiency is 40%, determine the power requirement.

Rolling

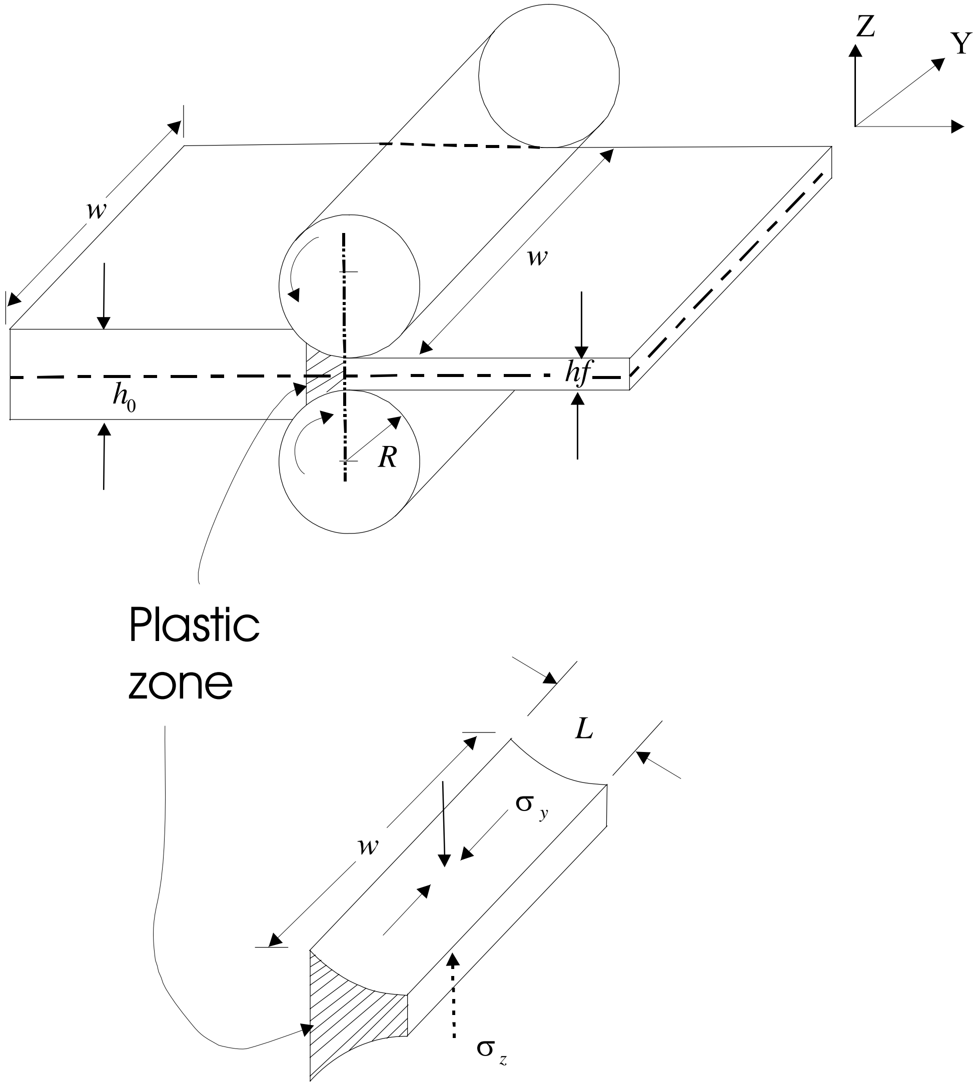
(Chapter 7, Lecture 16)

Perhaps the largest amount of metal is processed by rolling – a primary metal forming process for subsequent sheet forming (or stamping) operations.

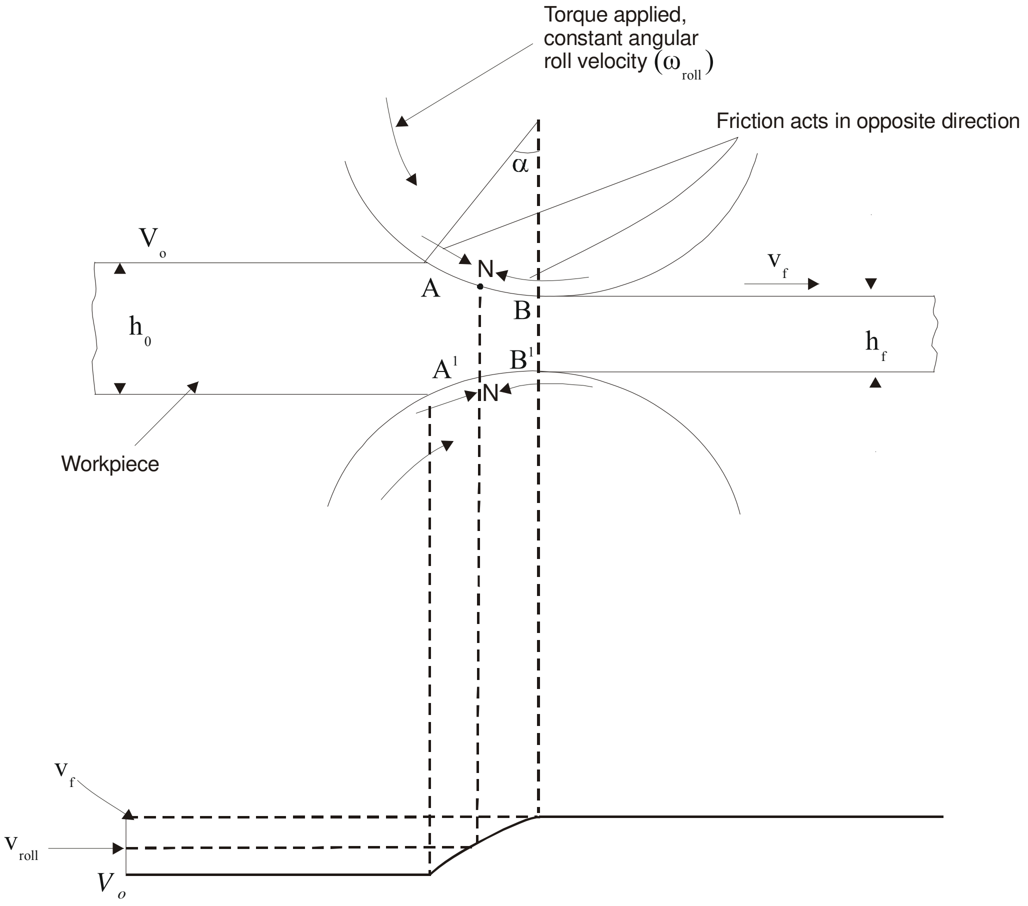


Flat Rolling

Rolling can be assumed as a plane strain compression problem, i.e., $\varepsilon_y \approx 0$ and $\varepsilon_z \approx -\varepsilon_x$ (except at the edge)



Flat Rolling



Neutral point, N ≈ a point on roll surface where the velocities of the roll and the metal are the same.

i.e., $v_{roll} = v_{metal}$

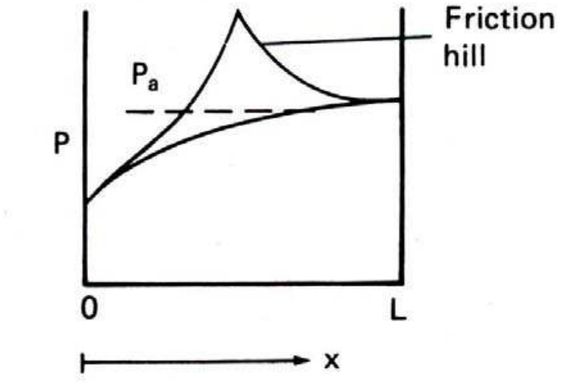
Left of N: $v_{metal} < v_{roll}$

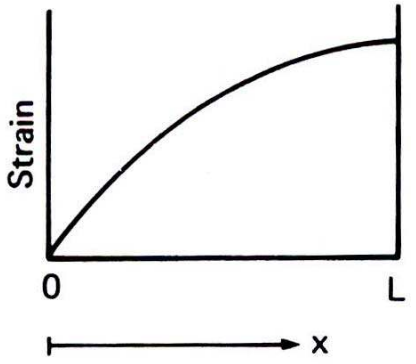
Right of N: $v_{metal} > v_{roll}$

Flat Rolling (Continued)

“Friction hill” exists - similar to plane strain compression of a block.

A schematic plot of roll pressure and strain distribution from entry to exit is shown below:

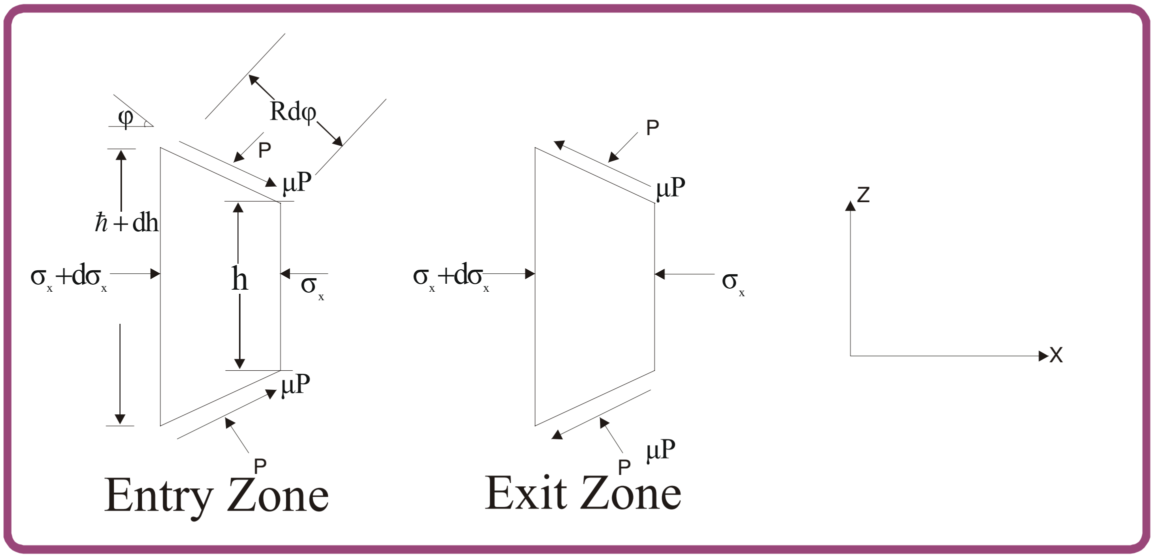




where L = Cord of arc of contact
(or “contact Length”, to be discussed later)

The peak in the roll pressure corresponds to the neutral point (to be discussed later)

Analysis of Flat Rolling - Slab Method



Entry Zone

Exit Zone

Assumptions

[1]. Plane strain deformation $(\varepsilon_w = 0)$ is assumed

[2]. Force balance equations are set-up slightly differently for entry/exit elements as shown.

[3]. For a strain hardening material, $Y = \bar{\sigma}(\bar{\varepsilon})$

[4]. Product of flow stress, Y, and current thickness, h, is constant, Yh ≈ constant

Analysis Steps for Roll Pressure Distribution

[1]. Force balance for the representative element in X and Z directions to obtain DE in σ_x and $\sigma_z \; (= -P)$

[2] . Simplify DE using the yield criterion

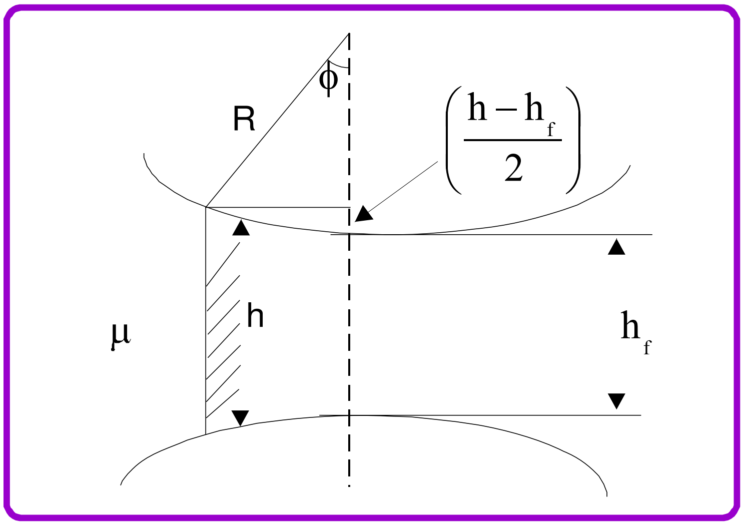
[3]. Simplify DE using following geometric relationships:

$$\cos\phi = \frac{R - \left[\dfrac{h - h_f}{2}\right]}{R} \qquad (1)$$

where R= radius of the rolls

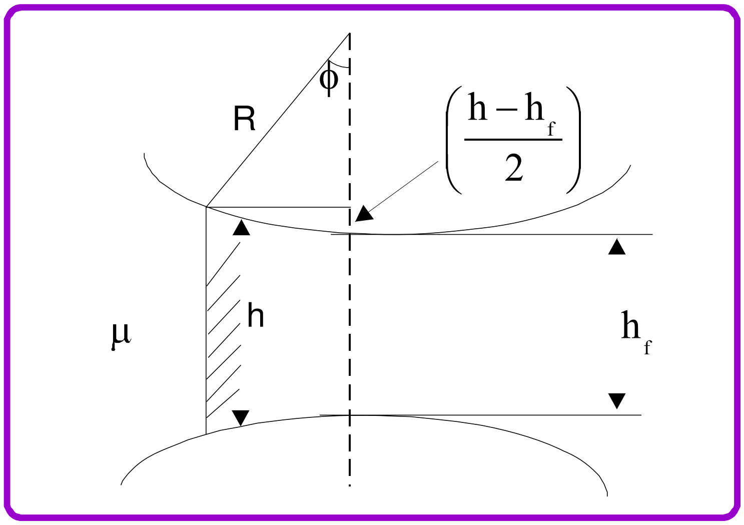
ϕ=angle between the roll and the work piece

$h,\ h_f$ = current and final thicknesses



Analysis Steps (continued)

$$\cos\phi = \frac{R - \left[\dfrac{h-h_f}{2}\right]}{R} \qquad (1)$$

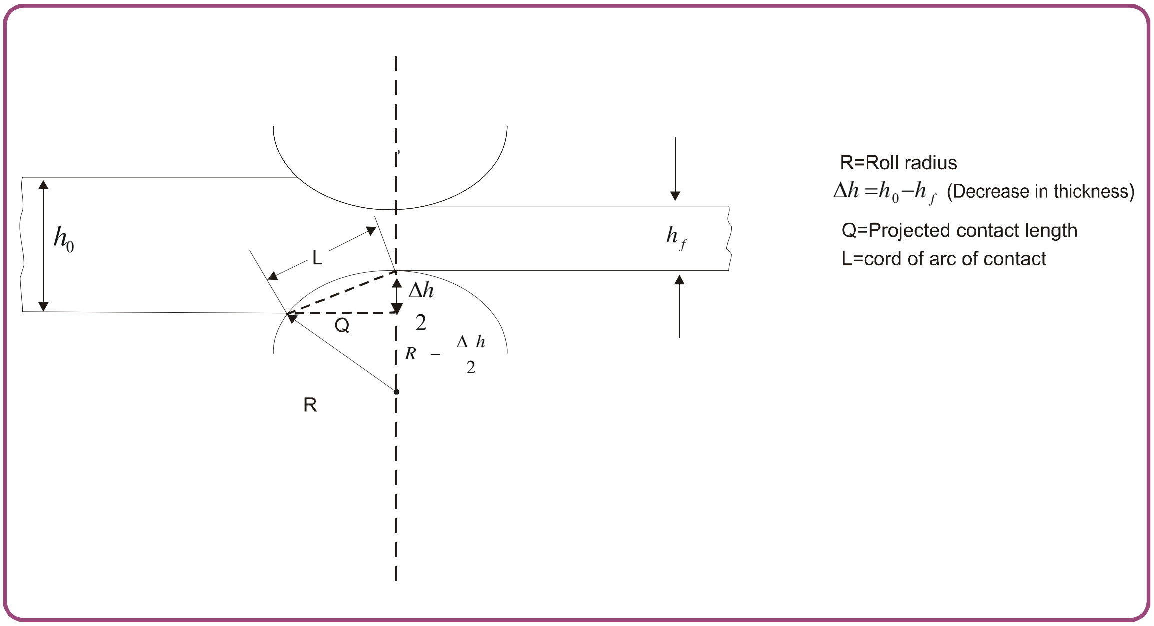


$$(1) \Rightarrow \qquad h = h_f + 2R(1-\cos\phi) \qquad (2)$$

Using $\cos\phi = 1 - \left(2\sin^2(\phi/2)\right)$, $\sin^2(\phi/2) \approx (\phi/2)^2$

$$(2) \Rightarrow \qquad h = h_f + R\phi^2 \qquad (3)$$

Flat Rolling - Roll Gap Geometry



Geometry: $$L^2 = Q^2 + \left(\frac{\Delta h}{2}\right)^2 \qquad \text{(A)}$$

and $$Q^2 = R^2 - \left(R - \frac{\Delta h}{2}\right)^2 = R\Delta h - \left(\frac{\Delta h}{2}\right)^2 \qquad \text{(B)}$$

$$\text{(B)} \rightarrow \text{(A)} \Rightarrow \quad L^2 = R\Delta h - \left(\frac{\Delta h}{2}\right)^2 + \left(\frac{\Delta h}{2}\right)^2$$

or $$L = \sqrt{R\Delta h} = \sqrt{Rrh_0} \qquad \text{(C)}$$

since rolling reduction, $r = \left(\Delta h / h_0\right)$, $r = \left(h_0 - h_f\right) / h_0$

$L =$ cord of arc of contact (“contact length”)

Roll Pressure Distribution

$$P = CY\frac{h}{R}\exp(\mp \mu H) \qquad (4)$$

where $$H \overset{def^{n}}{=} 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\phi\right) \qquad (5)$$

and C = constant (obtained from B. C.'s)

Notes:

[1]. The sign - and + in eqn. (4) applies from entry to neutral point and neutral to exit regions respectively.

[2]. H is a geometric parameter that varies with angle ϕ. The value of H corresponding to entry, neutral point are designated H_0 $(\phi = \alpha)$, H_n $(\phi = \phi_n)$ respectively.

Also, at exit, $H = H_f \overset{(5)}{=} 0$ $\left(\sin ce\ \phi = 0\right)$

[3]. The B. C. are different for entry and exit elements and therefore separate expressions are obtained for roll pressure distribution on both sides of the neutral point.

Roll Pressure Distribution - Entry Zone:

$$P = CY\frac{h}{R}\exp\left(-\mu H\right) \qquad (4)$$

where $$H \overset{def^n}{=} 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\phi\right) \qquad (5)$$

Calculation of C from B. C.'s:

$$\phi = \alpha, H = H_0 \overset{(5)}{=} 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\alpha\right), P = Y \quad (6)$$

$$(6) \rightarrow (4) \Rightarrow \quad \mathrm{C} = (R/\mathrm{h}_0)\exp\left(\mu \mathrm{H}_0\right) \qquad (7)$$

where h_0 = height of the work piece at the entry side

$(7) \rightarrow (4) \Rightarrow$

$$\mathrm{P} = Y\left(h/\mathrm{h}_0\right)\exp\left[\mu\left(H_0 - H\right)\right] \qquad (8)$$

Note: $\mathrm{P} = \mathrm{P}\left(\mathrm{Y}, \mu, \mathrm{h}_0, \mathrm{h}, \mathrm{R}, \mathrm{h}_\mathrm{f}, \alpha \text{ and } \phi\right)$

Roll Pressure Distribution - Exit Zone:

$$P = CY\frac{h}{R}\exp\left(+\mu H\right) \qquad (4)$$

$$\text{where} \quad H \overset{def^{n}}{=} 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\phi\right) \qquad (5)$$

Calculation of C from B. C.’s:

$$\phi = 0, \quad \mathrm{H} = \mathrm{H_f} \overset{(5)}{=} 0, \quad \mathrm{P} = \mathrm{Y} \qquad (9)$$

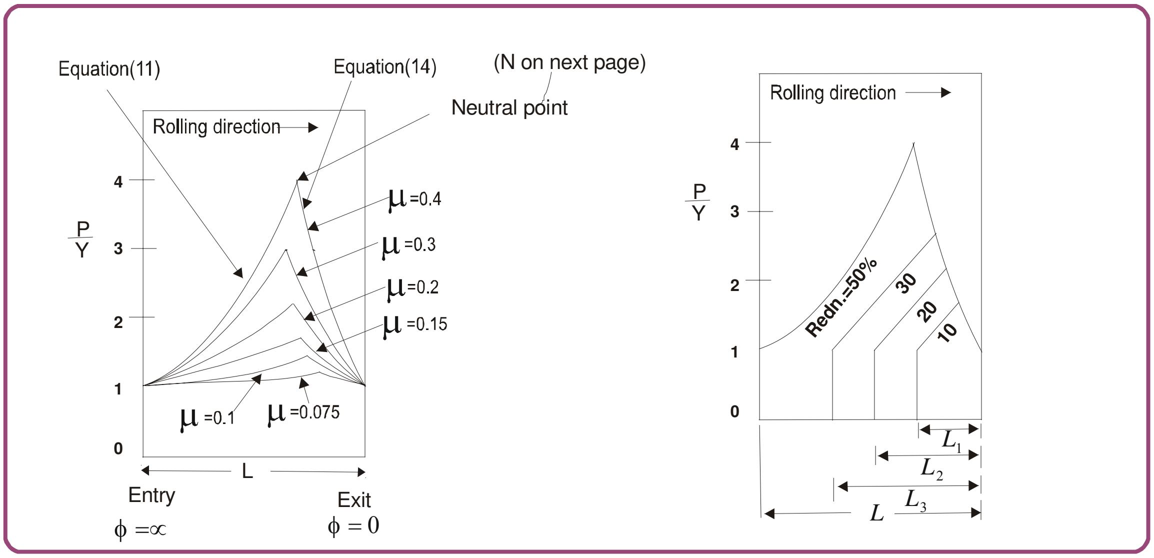
$$(9) \rightarrow (4) \Rightarrow \qquad C = R / h_f \qquad (10)$$

$$(10) \rightarrow (4) \Rightarrow \qquad \mathrm{P} = Y\left(\frac{h}{h_f}\right)\exp\left(\mu \mathrm{H}\right) \qquad (11)$$

Roll Pressure Distributions Vs. Contact Length:

Entry: $$P = Y\left(h/h_0\right)\exp\left[\mu\left(H_0 - H\right)\right] \qquad (8)$$

Exit: $$P = Y\left(\frac{h}{h_f}\right)\exp\left(\mu H\right) \qquad (11)$$



L =cord of arc of contact (“contact length”)

$$L = R\phi = R\sqrt{\frac{h-h_f}{R}} = \sqrt{R\left(h_0 - h_f\right)} = \sqrt{R\Delta h}$$

Remarks:

[1]. “Friction hill” is indicated by equations (8) and (11) is similar to the case of plane strain compression of a block.

[2]. Roll force can be reduced by lowering friction or smaller reductions. Reduction, r = $\left(h_0 - h_f\right)/h_0$

Example – Rolling

Commercial purity (1100) aluminum is routinely rolled in several passes but without annealing to a total reduction of over 98%. Typical n value and total elongation for 1100 Al are 0.25 and 35% respectively.

[1]. Find the reduction in area at fracture in a tensile test for this material?
[2]. Compare this to rolling reduction, and
[3]. Explain the reasons for the difference?

Example – Rolling

A strip is cold rolled on a tandem mill. Reductions are from 0.6 to 0.45 to 0.3 to 0.2 to 0.14.

[a]. Calculate the engineering and true strains for each pass?

[b]. Calculate the engineering and true strains in a single pass from 0.6 to 0.14.
[c]. Add up the engineering and true strains in [a].
[d]. Compare the strains from [b] and [c].

Flat Rolling (Lecture 17)

Determination of Neutral Point

Previous lecture gives roll pressure distribution as:

Entry: $$\mathrm{P} = Y\left(h/\mathrm{h}_0\right)\,\exp\left[\mu\left(\mathrm{H}_0 - H\right)\right] \qquad (8)$$

Exit: $$\mathrm{P} = Y\left(\frac{\mathrm{h}}{\mathrm{h}_\mathrm{f}}\right)\,\exp\left(\mu\mathrm{H}\right) \qquad (11)$$

where $$H \overset{def^n}{=} 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\phi\right) \qquad (5)$$

$$H_0 \overset{(5)}{=} 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\alpha\right),\ P = Y \qquad (\mathrm{A})$$

Equating (8) and (11) gives:

$$\frac{h_0}{h_f} = \frac{\exp\left[\mu\left(H_0 - H_n\right)\right]}{\exp\left(\mu H_n\right)} = \exp\left[\mu\left(H_0 - 2H_n\right)\right] \qquad (12)$$

or $$H_n \overset{(12)}{=} \frac{1}{2}\left(H_0 - \frac{1}{\mu}\ell n\frac{h_0}{h_f}\right) \qquad (13)$$

Flat Rolling - Determination of Neutral Point

$$H_n \overset{(12)}{=} \frac{1}{2}\left(H_0 - \frac{1}{\mu}\ell n \frac{h_0}{h_f}\right) \qquad (13)$$

By definition: $$H_n \overset{(5)}{=} 2\sqrt{\frac{R}{h_f}}\tan^{-1}\left(\sqrt{\frac{R}{h_f}}\ \phi_n\right) \qquad (14)$$

or $$\phi_n = \sqrt{\frac{h_f}{R}}\tan\left(\sqrt{\frac{h_f}{R}}\cdot\frac{H_n}{2}\right) \qquad (15)$$

Calculation procedure for obtaining ϕ_n:

$$R,\alpha,h_f \xrightarrow{(A)} H_0 \ \underset{(13)}{\overset{(\mu,h_0)}{\longrightarrow}}\ H_n \ \underset{(15)}{\longrightarrow}\ \phi_n$$

Flat Rolling – Front and Back Tension

[1]. Roll force can be reduced quite effectively by applying longitudinal tension.

[2]. Longitudinal tension can be applied at the entry (called back tension) or exit (called front tension) or both.

Equations (8) and (11) can be modified as :

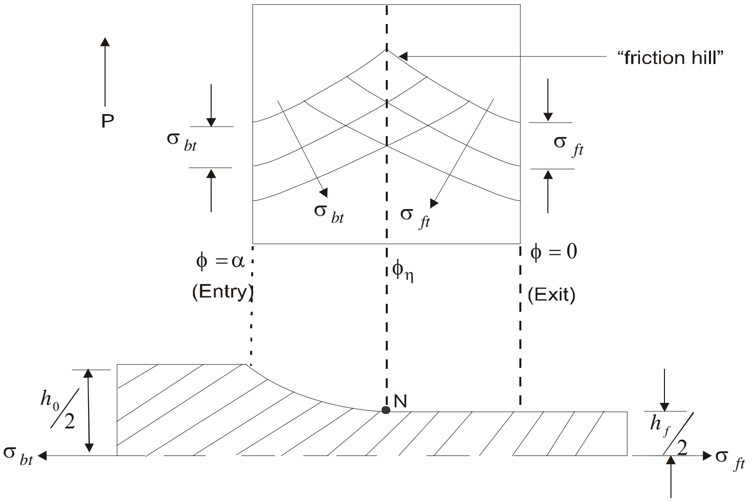
(8) ⇒ $$P = (Y - \sigma_{bt})\frac{h}{h_0}\ \exp\left\{\ \mu(H_0 - H)\right\} \qquad (16)$$

(11) ⇒ $$P = (Y - \sigma_{ft})\frac{h}{h_f}\ \exp\{\mu H\} \qquad (17)$$

(16), (17) ⇒

[a]. As $\sigma_{bt}\uparrow$, friction hill $\downarrow \Rightarrow P\downarrow \Rightarrow$ neutral point $\rightarrow$ exit

[b]. As $\sigma_{ft}\uparrow$, friction hill $\downarrow \Rightarrow P\downarrow \Rightarrow$ neutral point $\rightarrow$ entry



Average Pressure

If one assumes rolling = plane strain compression and neglect curvature, then for sliding friction :

From earlier plane strain compression of a block:

Compression block (= roll) pressure variation:

$$P/2k = \exp\left(2\mu x/h\right) \qquad (7\text{-}18)$$

Avg. block (= roll) pressure:

$$\frac{P_{av}}{2k} = \frac{h}{\mu b}\left[\exp\left(\frac{\mu b}{h}\right) - 1\right] \qquad (7\text{-}22)$$

On substituting, for flat rolling, the following:

$$b = L = \sqrt{R\Delta h} \text{ and } \quad h = (h_0 + h_f)/2 \qquad (B)$$

$$(B) \rightarrow (7\text{-}22) \Rightarrow \; P_{av} = \frac{\mathrm{h}}{\mu\mathrm{L}}\left(\exp\frac{\mu\mathrm{L}}{\mathrm{h}} - 1\right)\sigma_0 \qquad (7\text{-}41)$$

where average plane strain flow stress, $\sigma_0 = 2k$

Average Pressure with Back and Front Tension

$$P_{av} = \frac{h}{\mu L}\left(\exp\frac{\mu L}{h} - 1\right)\sigma_0 \qquad (7\text{-}41)$$

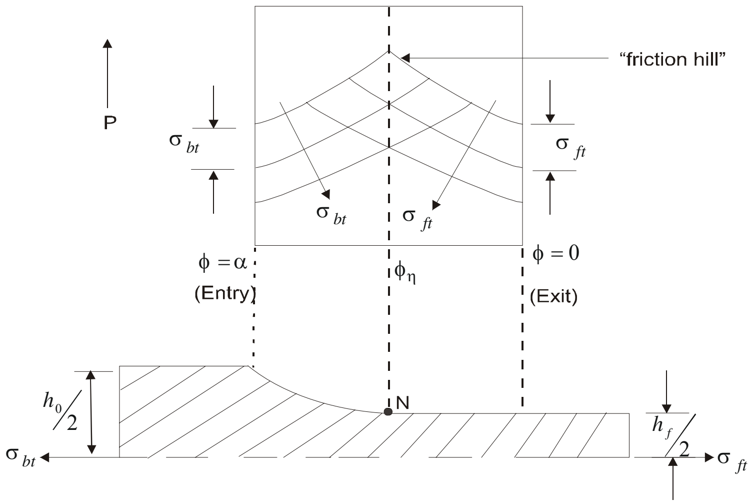
For work hardening material, $\sigma_0 = \left(\sigma_1 + \sigma_2\right)/2$

where σ_1 and σ_2: flow stress at the entrance and exit

If front and back tension, σ_{ft} and σ_{bt}, are applied:

$$\sigma_0 \to \sigma_0 - \sigma_t \qquad where \qquad \sigma_t = \frac{\sigma_{ft} + \sigma_{bt}}{2} \qquad (C)$$

(C) → (7-41) $\Rightarrow P_{av} = \frac{h}{\mu L}\left(\exp\frac{\mu L}{h} - 1\right)(\sigma_0 - \sigma_t)$ (7-42)



Determination of Rolling Process Parameters

Average Pressure:

$$P_{av} = \frac{h}{\mu L}\left(\exp\frac{\mu L}{h} - 1\right)(\sigma_0 - \sigma_t) \qquad (7\text{-}42)$$

Roll Force: $F = L \cdot W \cdot P_{av}$ $\quad (L \cdot W = \text{contact area})$

Torque Per Roll: $T = (FL/2)$

Power Reqd. Per Roll:

$$\text{Power} = T\omega = \left(\frac{FL}{2}\right)(2\pi N)$$

SI Units:

$$\text{Power} = \frac{\pi FLN}{60{,}000} \quad (kN \cdot m/\text{sec.}\ or\ kW)$$

where F in kN, L=meters, N= rpm

Imperial Units:

$$\text{Power} = \frac{\pi FLN}{33{,}000} \quad (\text{hp})$$

where F= kips, L=ft, N= rpm

Note: 1 kN.m/sec. = 1 kW = 1.81 hp

Roll Flattening

If the sheet is thin (small h), roll diameter is large (making L large), a large 'friction hill' results.

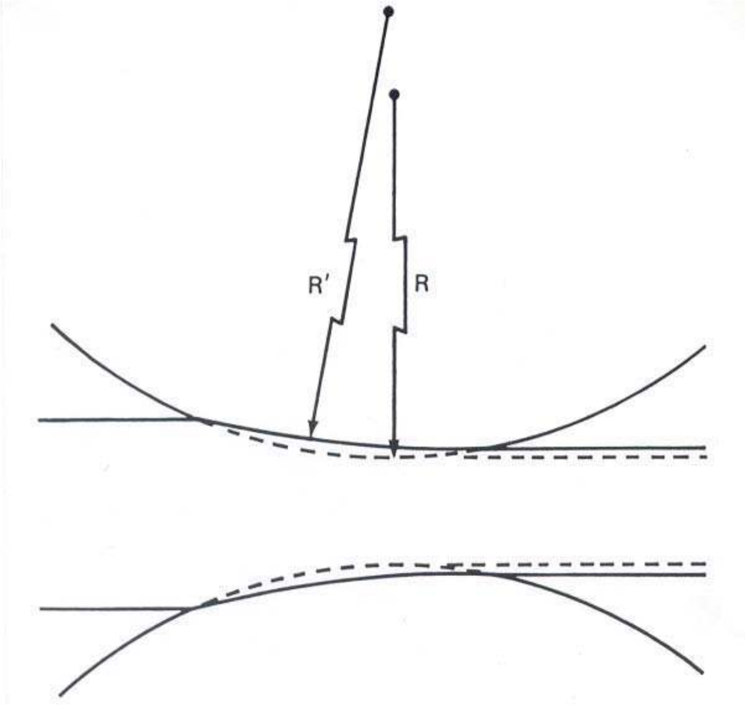
$\Rightarrow$ Larger roll pressure, P_{av}

$\Rightarrow$ Larger roll separating force/unit width,

$$F_s = \left(P_{av} \cdot L\right).$$

Consequence:

Flattening (elastic deflection) of the rolls.
The deflection of rolls puts a limit on minimum thickness of the sheet that can be rolled.



Roll Bending

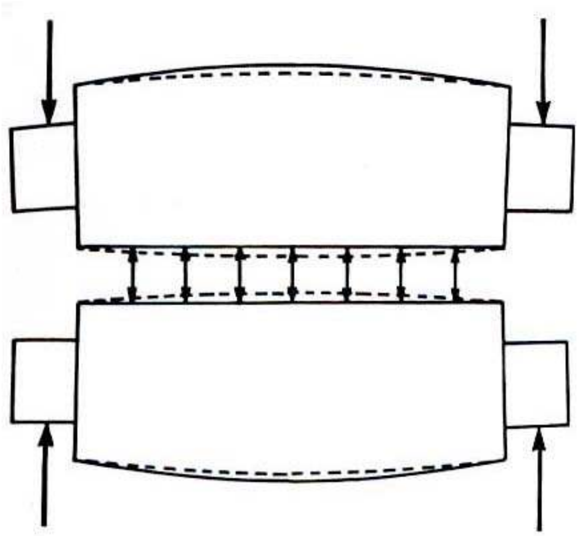
Small rolls when subjected to large separating forces cause roll bending.

Reduced by using:

[1]. back-up rolls

[2]. Roll material of high elastic modulus such as sintered carbide

[3]. cambered (or crowned) rolls

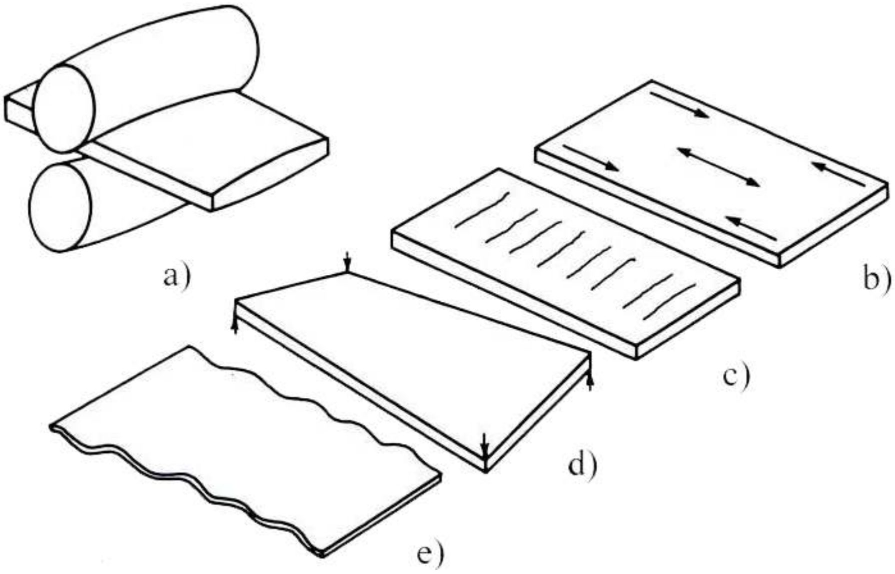


- - - - - - Roll profile with no load

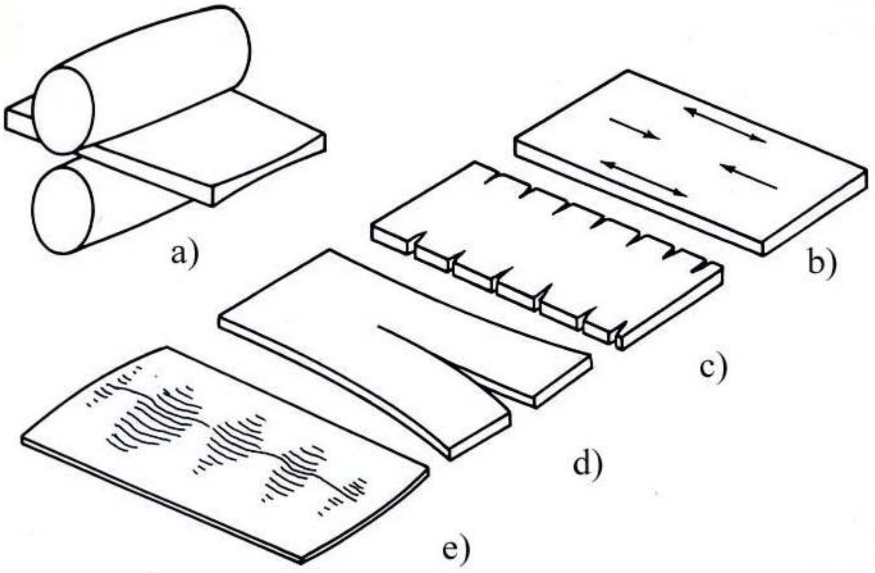
——— Roll profile with under load

Roll Camber

Insufficient camber results in centerline cracking, warping and edge wrinkling.



Over-cambered rolls can cause edge cracking, splitting and centre line wrinkling.



Example – Estimating Rolling Pass Schedule

Estimate rolling pass schedule for annealed copper and mild steel strips from 400 mm wide and 3.0 mm thick strips to 400 mm x 2.40 mm with 300 mm diameter rolls, if the roll load is not to exceed 2000 kN.

Given:

Material parameters for Cu:

K(copper)=315 MPa, n(copper)=0.55,

Material parameters for Steel:

K (mild steel) = 760 MPa, and n (mild steel)=0.19.

The coefficient of friction is given as 0.075.

Bending and Springback of a Work Hardening Material

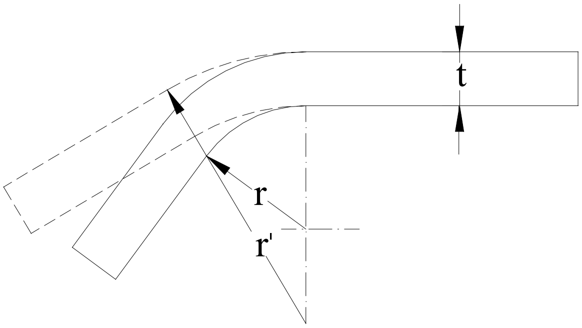
(Chapter 12, Lecture 20)

Power law, $\bar{\sigma} = K\bar{\varepsilon}^n$

and $$\sigma_x = K'\varepsilon_x^n = K'\left(\frac{z}{r}\right)^n \qquad \text{(A)}$$

where $$K' = K\left(\frac{4}{3}\right)^{(n+1)/2}$$

$K' \equiv$ plane strain strength coefficient
K and n come from uniaxial tension test.



(12-2): $$M = 2\int_0^{t/2} w\sigma_x z\,dz \overset{(A)}{=} 2\int_0^{t/2} wK'\left(\frac{z}{r}\right)^n z\,dz$$

or $$M = \left(\frac{2}{n+2}\right)\frac{wK'}{r^n}\left(\frac{t}{2}\right)^{n+2} \qquad \text{(12-9)}$$

Earlier, $$\Delta M = \frac{wE't^3}{12}\left(\frac{1}{r} - \frac{1}{r'}\right) \qquad \text{(12-6)}$$

Springback–Workhardening Material (continued)

Since moment under load, M, is equal to moment after springback ($M = \Delta M$):

$$(12-9),(12\text{-}6) \Rightarrow \left(\frac{2}{n+2}\right)\frac{wK'}{r^n}\left(\frac{t}{2}\right)^{n+2} = \frac{wE't^3}{12}\left(\frac{1}{r}-\frac{1}{r'}\right)$$

or
$$\left(\frac{1}{r}-\frac{1}{r'}\right) = \left(\frac{2\times 12}{2^2}\right)\left(\frac{1}{n+2}\right)\left(\frac{K'}{E'}\right)\left(\frac{t}{2r}\right)^n\left(\frac{t^2}{t^3}\right)$$

or
$$\left(\frac{1}{r}-\frac{1}{r'}\right) = \left(\frac{6}{n+2}\right)\left(\frac{K'}{E'}\right)\left(\frac{t}{2r}\right)^n\left(\frac{1}{t}\right) \qquad (12\text{-}10)$$

Earlier, for an elastic-perfectly plastic material:

$$\frac{1}{r}-\frac{1}{r'} = \frac{3\sigma_0}{tE'} \qquad (12\text{-}7)$$

$$\sigma_x' = \sigma_x - \Delta\sigma_x = \sigma_x - E'\Delta\varepsilon_x = \sigma_x - E'\left(\frac{z}{r}-\frac{z}{r'}\right)$$

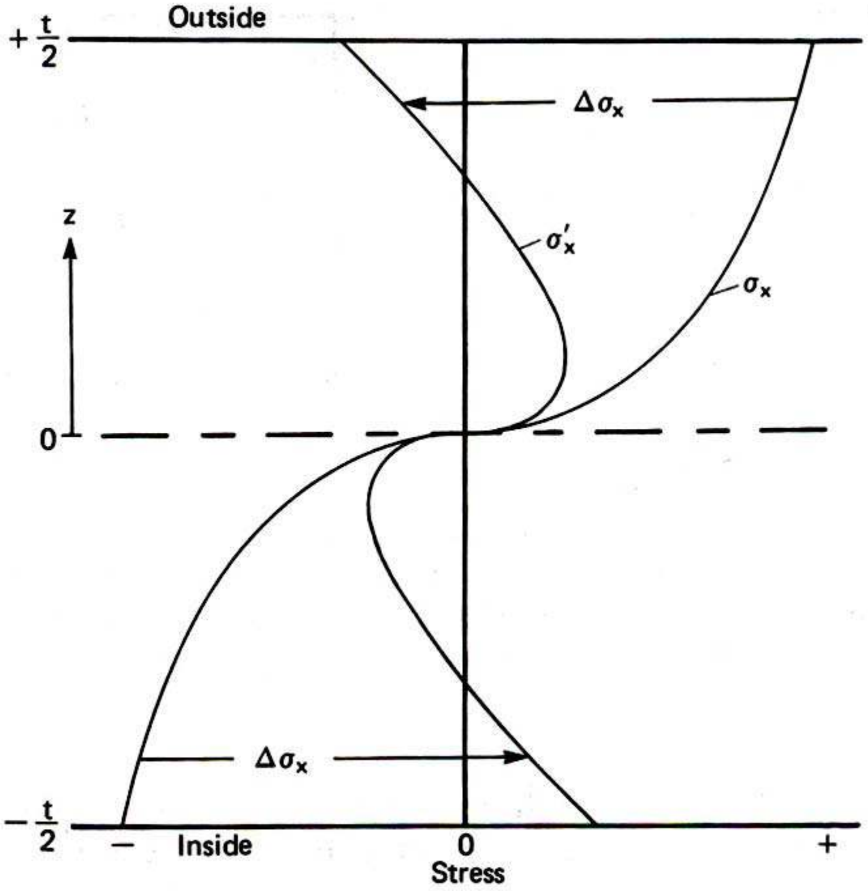
$$\sigma_x' \overset{(A)}{=} K'\left(\frac{z}{r}\right)^n - E'z\left[\left(\frac{6}{n+2}\right)\left(\frac{K'}{E'}\right)\left(\frac{t}{2r}\right)^n\left(\frac{1}{t}\right)\right]$$

or
$$\sigma_x' = K'\left(\frac{z}{r}\right)^n\left[1-\left(\frac{3}{n+2}\right)\left(\frac{2z}{t}\right)^{1-n}\right] \qquad (12\text{-}11)$$

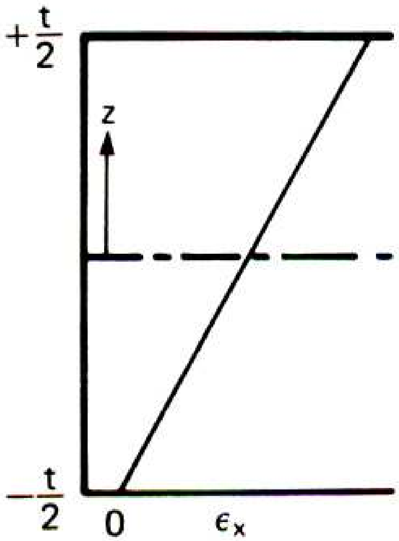
Springback–Work hardening Material(continued)

$$\sigma_x' = K'\left(\frac{z}{r}\right)^n\left[1-\left(\frac{3}{n+2}\right)\left(\frac{2z}{t}\right)^{1-n}\right] \quad (12\text{-}11)$$

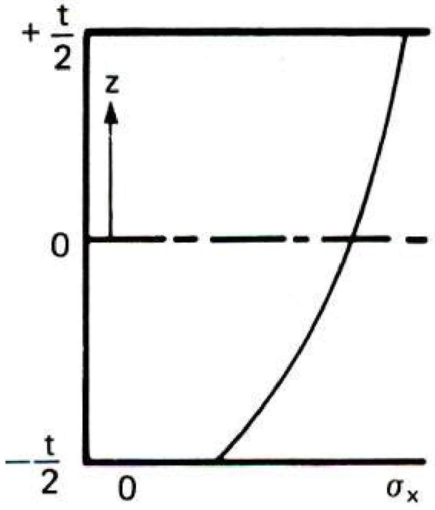
A plot of applied and residual stress distributions through the sheet thickness from equations (A) and (12-11) respectively is shown below:



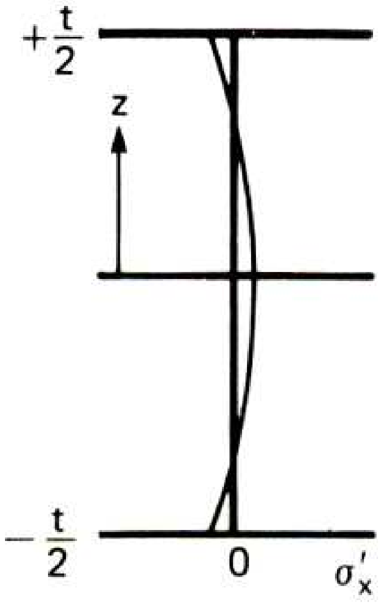
Bending with Superimposed Tension



strain distribution



stress distribution



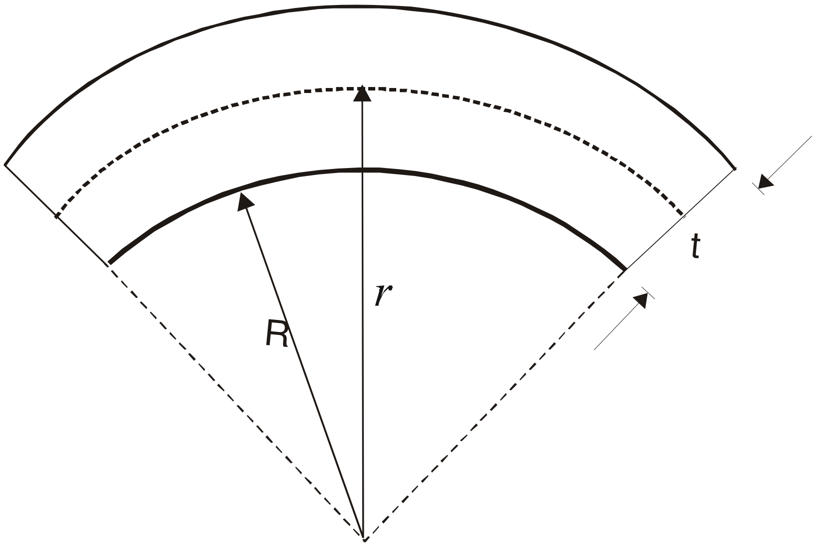
residual stress distribution

Sheet Bendability

Often measured by minimum R/t, i.e., minimum R value to which a sheet material can be bent for a given initial sheet thickness.

The lower the R/t value, the better is the 'bendability'

R = radius of the inner bend surface
t = initial sheet thickness



Note: R is also the radius of the bending tool.

Benadability and Reduction in Area at Fracture

$$\varepsilon_f \overset{defn.}{=} \ell n\left(\frac{A_0}{A_f}\right)$$

$$\Rightarrow \qquad \ell n\left(1+\frac{z}{r}\right) \overset{(12\text{–}1a)}{=} \ell n\left(\frac{A_0}{A_f}\right)$$

$A_0, A_f \equiv$ cross-sectional areas at start and at fracture

r = radius at the mid-plane

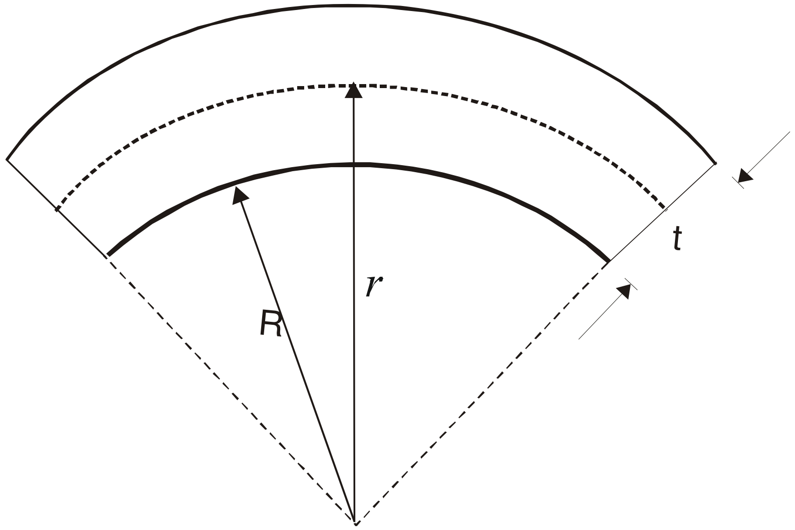
or $$1+\frac{z}{r} = \frac{A_0}{A_f} \Rightarrow \frac{r}{r+z} = \frac{A_f}{A_0}$$

or $$1-\frac{r}{r+z} = 1-\frac{A_f}{A_0} \Rightarrow \frac{z}{r+z} = A_r \qquad \text{(B)}$$

where $$A_r \overset{defn.}{=} \frac{A_0 - A_f}{A_0}$$

$A_r \equiv$ Reduction in area at fracture

Sheet Bendability (Continued)



$$\frac{r+z}{z} = \frac{r}{z} + 1 \overset{(B)}{=} \frac{1}{A_r} \qquad \text{(C)}$$

At outer (failure) surface ($z = \dfrac{t}{2}$):

$$z = \frac{t}{2} \overset{(C)}{\Rightarrow} 2\left(\frac{r}{t}\right) = \frac{1}{A_r} - 1 \qquad \text{(C)}$$

Also, from geometry, as shown above:

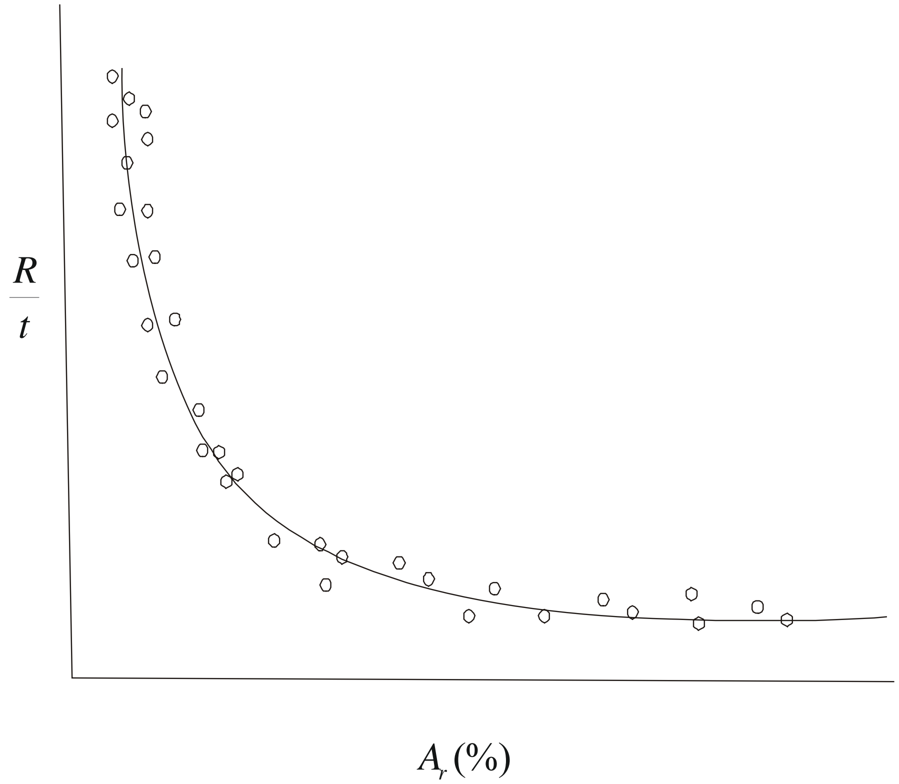
$$R = r - \frac{1}{2}t \quad \Rightarrow \quad r/t = R/t + 1/2 \qquad \text{(D)}$$

$$\text{(C),(D)} \Rightarrow \qquad \frac{R}{t} = \frac{1}{2A_r} - 1 \qquad \text{(12-13)}$$

Bendability and Reduction in Area at Fracture

$$\frac{R}{t} = \frac{1}{2A_r} - 1 \qquad (12\text{-}13)$$

A plot of equation (12-13) and some experimental Data for sheet metals:



Material with a higher A_r value (reduction in area at fracture)

$\Rightarrow$ Lower R/t value $\Rightarrow$ better bendability

Note: Equation (12-13) is not valid for very sharp radius bends due to shift in the natural axis.

Example: 12-5 (a) of text

Bending of a linear hardening material

It is necessary to bend an aluminum alloy sheet (0.040 in. thick) to a final radius of curvature of 3 in. The plastic portion of the stress-strain curve may be approximated by:

$$\sigma = \left(25 \times 10^3\right) + \left(25 \times 10^3\right)\ \varepsilon$$

and $E' = 11 \times 10^6$ psi

Accounting for springback, what radius of curvature must be designed into the tools if loading is pure bending ?